UNIVERSIDADE DE LISBOA
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MATHEMATICS COLLOQUIUM

Dia 12 de Outubro (terça-feira), às 18h15, na sala B3-01

"O-minimal ingredients in proofs of arithmetic conjectures in algebraic geometry"

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Abstract:

Recent work by Pila-Zannier and Pila yields proofs of some arithemtic conjecutres in algebraic geometry via connections to o-minimality, and in particular to a theorem of Pila and Wilkie about rational points on definable sets in o-minimal structures. The first paper, by Pila-Zannier, gave a new proof of the Manin-Mumford conjecture (Raynaud's theorem) about the torsion points on algebraic subvarieties of an abelian variety. Subsequent papers by Pila solved some open cases of the Andre-Oort conjecture. Beyond Pila-Wilkie's theorem, the connection to o-minimality requires the definability in o-minimal structures of certain transcendental holomorphic maps, such as various exponentials, the Weierstrass P-functions and the Riemann theta functions. In the compact case (e.g. for abelian varieties) this is immediate, but in the noncompact cases this definability needs to be established (see work with Starchenko).

In this talk I will try to explain the basic connections between the algebraic geometry problems and o-minimality, without going into too many technical details. I will mainly focus on the new proof to the Manin-Mumford conjecture.

The basic objects from algebraic geometry that I will discuss (and define) are: algebraic projective varieties, abelian varieties and their group operation. The analytic connection between complex tori and abelian varieties. The o-minimal structures I will discuss are: R_{an} (the expansion of the real field by the restriction of all real analytic functions to compact boxes in their domain) and R_{an,exp} (the expansion of R_{an} by the real exponential function). The notion of "definable sets" will be used extensively.

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