

Mês de:           **ABRIL 2014**

## SEMINÁRIO DE LÓGICA MATEMÁTICA

**Dia 3 de Abril (quinta-feira), às 15 horas, na Sala B3-01**

An axiomatic approach to modelling of orders of magnitude

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**Abstract:**

Many arguments deal informally with orders of magnitude of numbers. If one tries to maintain the intrinsic vagueness of orders of magnitude - they should be bounded, but stable under at least some additions -, they cannot be formalized with ordinary real numbers, due to the Archimedean property and Dedekind completion. Still there is the functional approach through *Oh's* and *oh's* and more generally Van der Corput's neutrices, both have some operational shortcomings.

Nonstandard Analysis disposes of a natural example of order of magnitude: the (external) set of infinitesimals is bounded and closed under addition. Adopting the terminology of Van der Corput, we call a *neutrix* an additive convex subgroup of the nonstandard reals. An *external number* is the set-theoretic sum of a nonstandard real and a neutrix. The external numbers capture the imprecise boundaries of informal orders of magnitude and permit algebraic operations which go beyond the calculus of the *Oh's* and *oh's*. This external calculus happens to be based more on semigroup operations than group operations, but happens to be fairly operational in concrete cases and allows for total order with a generalized form of Dedekind completion.

Based on joint work with Bruno Dinis, we discuss an axiomatics for the calculus of neutrices and external numbers, trying to do justice to the vagueness of orders of magnitude. In particular we consider foundational problems which appear due to the fact that some axioms are necessarily of second order, and the fact that the external calculus exceeds existing foundations for external sets.

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