

- ISABEL OITAVEM, *Recursion schemes for P, NP and Pspace*.

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$P$ ,  $NP$  and  $Pspace$  are well-known classes of computational complexity that can be described following different approaches. Here we describe them in a machine independent manner, using recursion schemes, which turn the known inclusions  $P \subseteq NP \subseteq Pspace$  obvious. This work contributes to a better understanding of the involved classes, but no separation result is foreseen.

Recursion-theoretic approaches lead to classes of functions instead of predicates (or boolean functions). Therefore, instead of  $P$  and  $Pspace$  we reach the classes  $Fptime$  and  $FPspace$ . As a class of functions corresponding to  $NP$  we choose  $Fptime \cup NP$ , and we adopt the notation  $FNptime$ .

Our strategy is, as always in recursion-theoretic contexts, to start with a set of initial functions — which should be basic from the complexity point of view — and to close it under composition and recursion schemes. The recursion schemes can be bounded or unbounded depending on the chosen approach. In the first case we consider the Cobham’s characterization of  $Fptime$  [2], in the second case we consider the Bellantoni-Cook characterization of  $Fptime$  [1]. In both cases we work over  $\mathbb{W}$ , instead of  $\mathbb{N}$ , where  $\mathbb{W}$  is interpreted over the set of 0-1 words.

We look to these three classes of complexity —  $Fptime$ ,  $FNptime$  and  $FPspace$  — as resulting from three different models of computation — deterministic, non-deterministic and alternating Turing machines — and imposing the same resource constraint (polynomial time). Thus the adopted recursion schemes should somehow reflect the “increasing” computational power of the computation model. For  $FNptime$ , besides the “calibration” of the recursion schemes, we have an additional problem since one is dealing with a class which is not closed under composition (because  $NP$  is not closed under negation). This work is based on [3] and [4].

[1] S. BELLANTONI AND S. COOK, *A new recursion-theoretic characterization of Polytime functions*, **Computational Complexity**, vol. 2 (1992), pp. 97–110.

[2] A. COBHAM, *The intrinsic computational difficulty of functions*, **Proc. of the 1964 International Congress for Logic, Methodology, and the Philosophy of Science**, ed. Y. Bar-Hillel, North Holland, Amsterdam (1965), pp. 24-30.

[3] I. OITAVEM, *Characterizing Pspace with pointers*, **Mathematical Logic Quarterly**, vol. 54 (2008), no. 3, pp. 317–323.

[4] ———, *A recursion-theoretic approach to NP*, **Annals of Pure and Applied Logic**, vol. 162 (2011), no. 8, pp. 661–666.