SEMINÁRIO DE LÓGICA MATEMÁTICA

Dia 17 de Outubro (quinta-feira), às 15 horas, na Sala B3-01

ON STRUCTURES WITH TWO SEMIGROUP OPERATIONS

Bruno Dinis

Abstract:

In the nonstandard framework the existence of infinitesimals is admitted, making it possible to have many different convex subgroups of $\mathbb{R}$. Such groups are called neutrices [6] [7]. In an axiomatic approach in which all infinite sets have nonstandard elements, such as HST [5], most neutrices are external sets. Simple examples are the external set of all infinitesimals, and the external set of all limited real numbers. External numbers (the sum of a real number with a neutrix) may be bounded without having an infimum and supremum, and may be invariant under at least some additions or translations, and therefore are models of orders of magnitude or transitions with imprecise boundaries.

It was shown in [4] that the class of external numbers equipped with addition and the class of external numbers which are not neutrices equipped with multiplication form commutative regular semigroups. Unlike real numbers, external numbers have individualized neutral and inverse elements for both addition and multiplication. It was also shown that the distributive law is valid under some restrictions that can be completely characterized. Moreover, the external numbers are totally ordered, even allowing for a sort of generalized completeness property [2] [1] [7]. Hence external numbers have to a large extent algebraic properties similar to those of real numbers. This justifies the introduction of common algebraic structures defined by axiomatic rules.

We show that ordered fields satisfy all the axioms presented with two exceptions and that the set of all cosets with respect to additive convex subgroups of an non-archimedean field (in particular the class of external numbers) satisfies all the axioms presented.

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[5] V. Kanovei, M. Reeken, *Nonstandard Analysis, axiomatically*, Springer Monographs in Mathematics (2004).

[6] F. Koudjeti, *Elements of External Calculus with an aplication to Mathematical Finance*, PhD thesis, Labyrinth publications, Capelle a/d IJssel, The Netherlands (1995).

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Local:
INSTITUTO PARA A INVESTIGAÇÃO
INTERDISCIPLINAR
Av. Prof. Gama Pinto, 2
1649-003 Lisboa

HOSP.
SANTA
MARIA

Av. Prof. Gama Pinto

Entrada

ENTRECAMPOS