UNIVERSIDADE DE LISBOA
CENTRO DE MATEMÁTICA E APLICAÇÕES FUNDAMENTAIS
Av. Prof. Gama Pinto 2, 1649-003 LISBOA, PORTUGAL Tel. (351) 217 904 700 FAX (351) 217 954 288

Mês de: Maio 2010

SEMINÁRIO DE LÓGICA MATEMÁTICA

Dia 20 de Maio (quinta-feira), às 18h, na Sala 3-10

"Kolmogorov Complexity and Entropy Measures"

ANDREIA TEIXEIRA

(SQIG - Instituto de Telecomunicações, Universidade do Porto)

Abstract:

Kolmogorov complexity was originated independently, in the 60s by Kolmogorov, Solomono and Chaitin. Informally, the Kolmogorov complexity of a string x is the length of the smallest program that generates x. Shannon entropy, $H()$, was introduced in 1948 by C.E. Shannon (1916-2001) and quanti es the uncertainty about the results of an experiment, i.e. it measures the number of bits necessary to describe an outcome from an event. Kolmogorov complexity and Shannon entropy are conceptually dif erent, as the former is based on the length of programs and the later in probability distributions. However, it is known that, for any recursive probability distribution, the expected value of the Kolmogorov complexity equals the Shannon entropy. Tsallis and Renyi entropies of order α (T_α and R_α , respectively) are two generalizations of Shannon entropy, such that for any probability distribution P , $T_1(P) = R_1(P) = H(P)$. We study the relationship between those three kinds of entropy with Kolmogorov complexity, showing that the expected value of the Kolmogorov complexity only equals Renyi and Tsallis entropies of order 1. We prove that the expected value of the time-bounded Kolmogorov complexity equals the Shannon entropy for distributions such that the cumulative probability distribution is computable in an alloted time. So, for these distributions, Shannon entropy is a function that captures the notion of computationally accessible information. An important constructive semi-measure based on Kolmogorov complexity is the universal semi-measure $m(x) = 2^{-K(x)}$, which dominates any other constructive semi-measure , in the sense that there is a constant $c_\mu = 2^{-K(\mu)}$ such that, for all x, $m(x) \geq c_\mu \mu\ (x)$. We study the Tsallis and Renyi entropies of the universal time-bounded distribution $m^t(x)$, proving that, for this distribution, both entropies converge if and only if $\alpha > 1$.

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