UNIVERSIDADE DE LISBOA
CENTRO DE MATEMÁTICA E APLICAÇÕES FUNDAMENTAIS
Av. Prof. Gama Pinto 2, 1649-003 LISBOA, PORTUGAL Tel. (351) 217 904 700 FAX (351) 217 954 288

Mês de: Julho 2010

SEMINÁRIO DE LÓGICA MATEMÁTICA

Dia 15 de Julho (quinta-feira), às 18h, na Sala 3-10

"Proof interpretations with truth"

JAIME GASPAR

(Technische Universitaet Darmstadt, Germany)

Abstract:

A proof interpretation I of a theory S in a theory T is a function mapping a formula A of S to a formula $A_I(x)$ of T (with distinguished variables x) verifying $S \vdash A \Rightarrow T \vdash A_I(t)$ for some terms t extracted from a proof of A. Proof interpretations are used in: (i) consistency results (e.g., if $\bot_I(x) = \bot$, then $S \vdash \bot \ \Rightarrow T \vdash \bot$, i.e., S is consistent relatively to T); (ii) conservation results (e.g., if $T \vdash A_I(x) \to A$ for Π^0_2 formulas A, then $S \vdash A \ \Rightarrow T \vdash A$, i.e., S is conservative over T for Π^0_2 formulas); (iii) unprovability results (e.g., if there are no terms t such that $T \vdash A_I(t)$, then $S \nvdash A$); (iv) closure under rules (e.g., if $S =T$, $(\exists x A(x))_I(x) = A_I(x)$ and $\mathsf{T} \vdash {A_I(x) \to A(x)}$, then $T \vdash \exists x A(x) \ \Rightarrow T \vdash A(t)$ for some term t, i.e., T has the existence property); (v) extracting computational content from proofs (e.g., extracting t in the previous point).
Closure under rules needs $T \vdash A_I(x) \to A$ that can be achieved by: (i) upgrading T to the so-called characterization theory that proves $\exists x A_I(x) \leftrightarrow A$; (ii) or hardwiring truth in I obtaining It verifying $T \vdash A_{It}(x) \to A$. The first option doesn't work if: (i) the characterization theory is classically inconsistent (e.g., some bounded proof interpretations); (ii) or we want applications to theories weaker than the characterization theory. So we turn to the second option and present a method to hardwire truth (in proof interpretations of Heyting arithmetic satisfying some mild conditions) by defining: (i) a function C that replaces each subformula A of a formula by $A \wedge A_c$ where A_c is a ``copy'' of $A$; (ii) an ``inverse'' function C^{-1} that replaces the ``copies'' $A_c$ by the ``originals'' A; (iii) $It = C^{-1} \circ I \circ C$. As examples we hardwire truth in: (i) modified realizability; (ii) Diller-Nahm functional interpretation; (iii) bounded modified realizability; (iv) bounded functional interpretation; (v) slash.
This is based on a joint work with Paulo Oliva.
References: Jaime Gaspar and Paulo Oliva. Proof interpretations with truth. Forthcoming in Mathematical Logic Quarterly.

Local:
INSTITUTO SUPERIOR TÉCNICO
Pavilhão de Matemática