UNIVERSIDADE DE LISBOA
CENTRO DE MATEMÁTICA E APLICAÇÕES FUNDAMENTAIS
Av. Prof. Gama Pinto 2, 1649-003 LISBOA, PORTUGAL Tel. (351) 217 904 700 FAX (351) 217 954 288

Mês de: DEZEMBRO 2013

SHORT COURSE (Two Sessions)

Dia 11 de Dezembro (quarta-feira) às 14:30h, na Sala B3-01

e

Dia 13 de Dezembro (sexta-feira) às 14:30h, na Sala B3-01

Sobolev Spaces

Gilles Lebeau

(IF e Universidade de Nice)

Let $s \in R$ be given. The aim of this mini course is to present some basic tools to answer the following question:

Let U be a subanalytic bounded subset of R^n. Does there exists a complex of spaces $\mathcal{H}^{s,j}(U), j \geq 0$ such that the following holds true:
-1. If U is Lipschitz, then the complex $\mathcal{H}^{s,\cdot}(U)$ is concentrate in degree 0 and is equal to a "classical" Sobolev space $H^s(U)$ (s derivatives in L^2).
-2. If U, V are two subanalytic bounded subsets of R^n, then one has the exact long Mayer Vietoris sequence, where $K^{j,s}(U) = H^j(\mathcal{H}^{s,\cdot}(U))$ is the cohomology of the complex $\mathcal{H}^{s,\cdot}(U)$

$$... \to K^{j,s}(U \cup V)) \to K^{j,s}(U) \oplus K^{j,s}(V) \to K^{j,s}(U \cap V)) \to K^{j+1,s}(U \cup V)) \to ...$$

A. We will first recall some basic definitions and properties of Sobolev spaces on open sets. In particular, we will show that for Lipschitz U, the classical definition of $H^s(U)$ has to be modified for $s = -1/2 - k, k \in N$. Then for Lipschitz U, V, such that $U \cup V$ and $U \cap V$ are Lipschitz, we will present a proof of the exactness of the short exact sequence

$$0 \to H^s(U \cup V) \to H^s(U) \oplus H^s(V) \to H^s(U \cap V) \to 0$$

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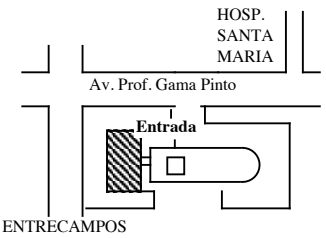

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B. We will show how to extend the category of Hilbert spaces to get a suitable abelian category $Abel - \mathcal{H}$ such that the spaces $\mathcal{H}^{s,j}(U)$ are objects of this category. Then following an idea of S. Guillermou and P. Schapira, we will explain how to construct the complex $\mathcal{H}^{s,j}(U)$ for a subanalytic U and $s \leq 0$.

C. Finally, we will explicitly compute the spaces $\mathcal{H}^{s,j}(U)$ for any $s \in R$ when U is a subanalytic subset of R^2.

References:

-1. S. Guillermou and P. Schapira. *Subanalytic topologies I: Construction of sheaves.* arXiv:1212.4326v2.

-2. M. Kashiwara and P. Schapira. *Sheaves on Manifolds*, Grundlehren 292, Springer Verlag (1990)

-3. J.L. Lions and E. Magenes. *Problèmes aux limites non homogènes et applications*, Vol 1 et 2, Travaux et recherches mathématiques, Dunod (1968)

-4. J.-P. Schneiders *Quasi-Abelien Categories and Sheaves*, Mém.Soc.Math.France 76, (1999)

-5. E. Stein. *Singular integrals and differentiability properties of functions*, Mathematical Series, n.30, Princeton University Press (1970)

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