Parabolic Equations with Double Variable Nonlinearity

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We review the recent results on the existence, nonexistence and uniqueness of solutions to the homogeneous Dirichlet problem for the doubly nonlinear parabolic equations of the form

$$\frac{d}{dt}\left(|u|^{m(z)-1}u\right) = \sum_{i=1}^{n} D_i\left(|D_i u|^{p_i(z)-2}D_i u\right) + g(z,u), \tag{1}$$

$$u_t = \operatorname{div}\left(|u|^{\alpha(z)}|\nabla u|^{p(z)-2}\nabla u\right) + f(z,u), \tag{2}$$

where $z=(x,t)$ denote the points of the cylinder $Q=\Omega\times(0,T)$, p_i, m, α and σ are given functions of the argument z. The following issues are discussed:

- conditions on the monotonicity and regularity of the exponents p_i, m, α, σ which guarantee the existence of weak solutions of equation (2), or strong solutions of the anisotropic equation (1) in the space

$$\mathcal{V} = \left\{u:\ u\in L^\infty(Q),\ |D_{x_i}u|^{p_i(z)}\in L^\infty(0,T;L^1(\Omega)),\ |u|^{m(z)-1}u_t^2\in L^1(Q)\right\};$$

- energy estimates for weak and strong solutions;
- comparison and uniqueness theorems for the isotropic equation (1) under the additional assumptions on the regularity of the solution: $\partial_t\left(|u|^{m(z)-1}u\right)\in L^1(Q)$;
- global boundedness versus finite time blow-up.

Part of the results can be found in the papers [1-5].

Joint work with S. Antontsev.

References

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