A class of electromagnetic p(x,t)-curl systems: existence and uniqueness, blow-up and finite time extinction

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We consider the following class of $p(x,t)$-curl systems arising in electromagnetism

$$\partial_t \boldsymbol{h} + \nabla \times \left(|\nabla \times \boldsymbol{h}|^{p(x,t)-2} \nabla \times \boldsymbol{h} \right) = \boldsymbol{f}(\boldsymbol{h}), \quad \nabla \cdot \boldsymbol{h} = 0 \quad \text{in } Q_T, \tag{1}$$

$$|\nabla \times \boldsymbol{h}|^{p(x,t)-2} \nabla \times \boldsymbol{h} \times \boldsymbol{n} = \boldsymbol{0}, \quad \boldsymbol{h} \cdot \boldsymbol{n} = 0 \text{ on } \Sigma_T, \tag{2}$$

$$\boldsymbol{h}(\,\cdot\,, 0) = \boldsymbol{h}_0 \text{ in } \Omega. \tag{3}$$

where $\boldsymbol{h}$ is the unknown magnetic field and $\boldsymbol{h}_0$ is a given function. The given log -continuous function $p(x,t)$ satisfies $\frac{6}{5} < p^- \leq p(x,t) \leq p^+ < \infty$. The nonlinear function $\boldsymbol{f}(\boldsymbol{h})$ can model either a source term of the type $\boldsymbol{f}(\boldsymbol{h}) = \boldsymbol{h} \left(\int_\Omega |\boldsymbol{h}|^2 \right)^{\frac{\sigma-2}{2}}$, with $\sigma > 1$ or a sink term $\boldsymbol{f}(\boldsymbol{h}) = -\boldsymbol{h} \left(\int_\Omega |\boldsymbol{h}|^2 \right)^{-\lambda}$, with $\lambda > 0$.

After establishing the adequate functional framework for the weak formulation of the problem and proving existence of a suitable basis for this functional space, we use Galerkin's method to obtain existence of solution.

Blowup of local solutions is studied in the case of source term and finite time extinction of solution is proved in the case of sink term. Analogous results for system (1)-(3) with constant exponent p were proved in [1].

(Joint work with Stanislav Antontsev and Fernando Miranda).

References

1. Antontsev S., Miranda F., Santos L., A class of electromagnetic p-curl systems: blow-up and finite time extinction, Nonlinear Analysis: Theory, Methods, Applications, 75(2012), pp.3916-3929.