Singular solutions to the Navier-Stokes equations

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We consider stationary solutions of the Navier-Stokes equations having singularities in flow domain or on its boundary. This class contains the well-known self-similar solutions obtained by Jeffery, Hamel, Landau, Goldshtik. Later these solutions were studied by Serrin, Akulenko et al, Aristov, Rivkind and Solonnikov, Sverak and author. Situation, when stationary self-similar solution is not unique in one range of governing parameters and does not exist in another range, is typical for solutions of such class. We demonstrate both possibilities on examples of solutions describing free boundary motions.

Second part of communication contains author's original results on the study of motions in plane or axially symmetric domains in the presence of singularities like a source or a drain. This problem is the limiting case of the famous Leray problem concerning viscous flows in domains with a multiply connected boundary. The velocity field in singular analogues of the Leray problem has an infinite Dirichlet integral. Sufficient conditions for solvability of the problems based on new a priori estimates of the regular part of solution are formulated.

In third part, we formulate a number of open questions in this area. One of them is 2D problem about the point vortex in domain bounded by a solid wall. Its axially symmetric analogue is more difficult due to intersection of a straight vortex line with a smooth solid surface of revolution. We suppose that the Goldshtik solution gives the main term of its solution near intersection points.

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