Finite Element Method for Equations and Systems of reaction diffusion type

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Abstract

In this work we study parabolic problems with free boundaries.

First we consider the following semilinear parabolic equation, with variable exponent of nonlinearity, which appears in continuum mechanics.

$$u_t = div(|u|^{\gamma(\mathbf{x})}\nabla u) + f(\mathbf{x}, t), \quad \mathbf{x} \in \Omega \subset R^2, t \in]0, T]$$

Since the problem may be of degenerate type, we utilize an approximate problem, regularized by introducing a parameter ε. We prove, under certain conditions on γ and f, that the weak solution of the approximate problem converges to the weak solution of the initial problem, when the parameter ε tends to zero. Discrete solutions are built using the FEM and the convergence of these for the weak solution of the approximate problem is proved.

Next we study a system of parabolic equations with nonlocal nonlinearity, appearing in biological and ecological systems, of the type

$$\begin{cases} u_t - a_1(l_1(u), l_2(v))\Delta u + |u|^{p-2}u = 0, & \text{in} \quad \Omega\times]0,T] \\ v_t - a_2(l_1(u), l_2(v))\Delta v + |v|^{p-2}v = 0, & \text{in} \quad \Omega\times]0,T] \\ u(x,t) = v(x,t) = 0 & \text{on} \quad \partial\Omega\times]0,T] \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x) & \text{in} \quad \Omega \end{cases}.$$

We prove the existence, uniqueness and the asymptotic behavior of strong global solutions. The order of convergence of a linearized Euler-Galerkin Finite Element Method is obtained.

For each problem, we present some numerical results of a MatLab implementation of the method.

Joint work with Professors Rui Almeida, Stanislav Antontsev and Jorge Ferreira.

References

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[2] F.J.S.A. Corrêa, Silvano D.B. Menezes, and J. Ferreira. On a class of problems involving a nonlocal operator. *Applied Mathematics and Computation*, 147(2):475 – 489, 2004.