GLOBAL L_2–SOLVABILITY OF A PROBLEM ON THE MOTION OF A DROP IN AN INCOMPRESSIBLE FLUID WITHOUT SURFACE TENSION

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Unsteady motion of two viscous incompressible fluids is considered in a bounded domain with a solid boundary. We assume the fluids to be immiscible and situated one in another. The liquids are separated by an unknown interface on which surface tension is neglected. On the outer surface, the non-slip condition is supposed. This motion is governed by an interface value problem for the Navier-Stokes system. We prove that this problem is uniquely solvable in Sobolev – Slobodetskiĭ spaces for every positive time interval if the data are small enough.

We give a mathematical formulation of the problem.

Let, at the initial moment $t = 0$, a fluid with the viscosity $\nu^+ > 0$ and the density $\rho^+ > 0$ occupy a bounded domain $\Omega_0^+ \subset \mathbb{R}^3$, $\Gamma_0 \equiv \partial\Omega_0^+$. And in the "exterior" domain Ω_0^- which surrounds Ω_0^+, there let be a fluid with the viscosity $\nu^- > 0$ and the density $\rho^- > 0$. The boundary $S \equiv \partial(\Omega_0^+ \cup \Gamma_0 \cup \Omega_0^-)$ is a given closed surface, $S \cap \Gamma_0 = \emptyset$.

For every $t > 0$, it is necessary to find the interface Γ_t between the domains Ω_t^+ and Ω_t^-, as well as the velocity vector field $\boldsymbol{v}(x,t) = (v_1, v_2, v_3)$ and the pressure function p of both fluids satisfying the following initial-boundary value problem:

$$\begin{aligned}
&\mathcal{D}_t\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \nu^\pm\nabla^2\boldsymbol{v} + \frac{1}{\rho^\pm}\nabla p = \boldsymbol{f}, \quad \nabla\cdot\boldsymbol{v} = 0 \quad \text{in} \quad \Omega_t^- \cup \Omega_t^+, \quad t > 0,\\
&\boldsymbol{v}|_{t=0} = \boldsymbol{v}_0 \quad \text{in} \quad \Omega_0^- \cup \Omega_0^+, \qquad\qquad (1)\\
&[\boldsymbol{v}]|_{\Gamma_t} = \lim_{\substack{x\to x_0\in\Gamma_t,\\ x\in\Omega_t^+}} \boldsymbol{v}(x) - \lim_{\substack{x\to x_0\in\Gamma_t,\\ x\in\Omega_t^-}} \boldsymbol{v}(x) = 0, \quad [\mathbb{T}\boldsymbol{n}]|_{\Gamma_t} = 0, \quad \boldsymbol{v}|_S = 0.
\end{aligned}$$

Here $\mathcal{D}_t = \partial/\partial t$, $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$, $\nu^\pm, \rho^\pm$ are the step functions of viscosity and density, respectively, $\boldsymbol{f}$ is the given vector field of mass forces, $\boldsymbol{v}_0$ is the initial velocity, $\mathbb{T}$ is the stress tensor with the elements $T_{ik} = -\delta_i^k p + \mu^\pm(\partial v_i/\partial x_k + \partial v_k/\partial x_i)$, $i,k = 1,2,3$; $\mu^\pm = \nu^\pm\rho^\pm$, δ_i^k is the Kronecker symbol, $\boldsymbol{n}$ is the outward normal to Ω_t^+. The centered dot denotes the Cartesian scalar product.

Local (in time) existence theorem was established for problem (1) in [1, 2].

For a sufficiently small initial velocity vector field $\boldsymbol{v}_0$ and mass force $\boldsymbol{f}$, we prove the existence of a unique solution to the problem on an infinite time interval. The proof is based on the exponential energy estimate of the solution for every sufficiently small time interval.

References

[1] I.V.Denisova, The motion of a drop in a fluid flow, *Dinam. sploshnoi sredy*, SOAN SSSR, Novosibirsk, **93/94** (1989), 32–37 (in Russian).

[2] I. V. Denisova, Problem of the motion of two viscous incompressible fluids separated by a closed free interface, *Acta Appl. Math.* **37** (1994), 31–40.