Complex Ginzburg-Landau equation with absorption: existence, uniqueness and localization properties

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Let $\Omega \subset \mathbf{R}^n$ be a bounded domain with Lipschitz-continuous boundary Γ and $Q_T = \Omega \times (0, T]$. We consider the following initial boundary value problem for complex function $u = \mathrm{Re}\ u + i\,\mathrm{Im}\,u$

$$e^{-i\gamma} u_t = \triangle u - a(x,t)\,|u|^{\sigma-2}\,u + f(x,t),\ 1 < \sigma < 2, -\pi/2 < \gamma < \pi/2, \quad (1)$$

$$u(x,0) = u_0(x),\ x \in \Omega;\ u|_{\Gamma_T} = 0,\ \Gamma_T = \partial\Omega \times (0,T). \quad (2)$$

Here a, f and u_0 are given complex value functions such that

$$0 \le |a| < \infty, f \in L^2(Q_T), u_0 \in L^2(\Omega).$$

Usually (1) is called as the time dependent complex Ginzburg-Landau (CGL). CGL with $\gamma = 0$ reduces to the well-known nonlinear heat equation $u_t - \Delta u = -|u|^{\sigma-2}u$. For $\gamma = \pm\pi/2$ CGL becomes the well-known nonlinear Schröodinger equation. CGL is very important as a model for the study of the pattern formation and the onset of instabilities in non equilibrium fluid dynamic systems. The mathematical study of the time dependent CGL equation can be found in many papers (see for example [2]). In this talk we prove existence of a global weak solution for (1),(2) applying the Galerkin approximation method. Unfortunately, we cannot prove a uniqueness result for the weak solutions. We prove also some localization properties for the solutions and finite time extinction in special cases. Part of the results can be found in [1].

Joint work with J. P. Dias and M. Figueira.

References

[1] S. N. Antontsev, J. P. Dias and M. Figueira. Complex Ginzburg-Landau equation with absorption: existence, uniqueness and localization properties. Preprint-004(2013), CMAF, University of Lisbon, pp.1-16.

[2] T. Cazenave, F. Dickstein, and F. B. Weissler, *Finite-Time Blowup for a Complex Ginzburg–Landau Equation*, SIAM J. Math. Anal., 45 (2013), pp. 244–266.