Titles and Abstracts

Computable objects in D-module theory

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D-modules theory studies modules over the ring D of linear differential operators. For computational purposes we can restrict ourselves to the case of finitely generated D-modules over the complex Weyl algebra $D = A_n$. Elements in the Weyl algebra A_n are linear differential operators with coefficients in the polynomial ring $\mathbb{C}[x]$ of complex polynomials in n variables $x = (x_1, ..., x_n)$. The Weyl algebra A_n is (for $n > 0$) a non-commutative noetherian ring. For a given non-zero polynomial f, the ring of rational functions $\mathbb{C}[x]_f$ is finitely generated considered as a module over A_n (this main result is due to J. Bernstein). Finding a system of generators of such a module is a computationally difficult task and it is related to the computation of the annihilating ideal of the rational function $1/f$. This annihilating ideal can be computed by using algorithms of T. Oaku and N. Takayama. These algorithms use Groebner basis computations in the Weyl algebra A_n. In this talk we will describe the role of logarithmic D-modules in the "logarithmic comparison conjecture", an open problem concerning both the annihilating ideal of $1/f$ and the comparison between the cohomology of the meromorphic and the logarithmic de Rham complexes associated to the polynomial f.

On irregular binomial D-modules

María-Cruz Fernández-Fernández
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This is a joint work with Francisco-Jesús Castro-Jiménez [2]. Binomial D-modules have been introduced by Dickenstein, Matusevich and Miller [1]. These authors have answered essential questions about binomial D-modules. In particular, they characterized the holonomicity of binomial D-modules and studied their holomorphic solution space around a non singular point. Let us fix a matrix $A = (a_{ij}) \in \mathbb{Z}^{d\times n}$ whose columns $a_1, \cdots, a_n$ span the lattice $\mathbb{Z}^d$. We also assume that all $a_i \neq 0$ and that the cone generated by the columns in $\mathbb{R}^d$ contains no lines (one says in this case that this cone is pointed). The matrix A induces a $\mathbb{Z}^d$-grading on the Weyl algebra D (also called the A-grading) by defining $\deg(\partial_i) = -a_i$ and $\deg(x_i) = a_i$.

A binomial D-module $M_A(I, \beta)$ is determined by a triple (A, β, I) where β is a vector in $\mathbb{C}^d$ and I is an A-graded binomial ideal $I \subset \mathbb{C}[\partial_1, \cdots, \partial_n]$. If I is the toric ideal associated with A then $M_A(I, \beta)$ is nothing but a hypergeometric D-module as introduced by Gelfand, Graev, Kapranov and Zelevinsky in the late 80's.

In this talk, we will provide a characterization for the regularity of holonomic binomial D-modules and we will see some examples illustrating this result and exhibiting new behavior that is forbidden to hypergeometric D-modules. We will also describe some other aspects of the irregularity of a binomial D-module $M_A(I, \beta)$ in terms of known results for the hypergeometric systems corresponding to certain associated primes of I that depend on β.

References

[1] A. Dickenstein, L. Matusevich, E. Miller. *Binomial D-modules.* Duke Math. J. 151 no. 3 (2010), 385-429.
[2] M.C. Fernández-Fernández and F.J. Castro-Jiménez. *On irregular binomial D- modules.* arXiv:1012.0618v2 [math.AG].

Higher Dimensional Rigid Local Systems I

Orlando Neto
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We present a generalization of notion of rigid local system due to Nicholas Katz. Our notion is deeply related to the classical notion in the case that the local system is defined on the complementary of a weighted homogeneous hypersurface. Moreover, it generalizes a notion of system of PDE's without accessory parameters due to Sato, Kashiwara, Kimura and Oshima. This is a joint work with Pedro Silva.

Sheaves on subanalytic sites and D-modules

Luca Prelli
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Sheaf theory is not well suited to study objects which are not defined by local properties. It is the case, for example, of functional spaces with growth conditions. Since the study of the solutions of a system of PDE in these spaces is of great importance (solutions of irregular D-modules, Laplace transform, etc.), in 2001 Kashiwara and Schapira introduced the subanalytic site and proved that some of this spaces can be realized as sheaves on a subanalytic site. In this talk we will recall the theory of subanalytic sheaves and give an overview of some recent developments and applications.

Closed Differential Forms on Moduli Spaces of Sheaves

Francesco Bottacin
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Geometric properties of moduli spaces of sheaves over a smooth projective variety (or Kähler manifold) X are often induced by similar properties of X itself. A beautiful example of this general fact was discovered, more then 25 years ago, by Mukai: if X is an algebraic surface endowed with a holomorphic symplectic structure, then a holomorphic symplectic structure can also be constructed on moduli spaces of stable sheaves on X.

After reviewing Mukai's construction, we shall see how it fits in a much more general framework. Let X be a smooth n-dimensional projective variety, and let Y be a moduli space of stable sheaves on X. By using the local Atiyah class of a universal family of sheaves on Y we shall construct natural maps

$$f : H^i(X, \Omega_X^j) \longrightarrow H^{k+i-n}(Y, \Omega_Y^{k+j-n}),$$

for any $i, j = 1, \dots, n$ and any $k \geq \max\{n-i, n-j\}$. This gives us a natural way to construct closed differential forms on moduli spaces of sheaves. As an application, we shall describe the construction of closed differential forms on the Hilbert schemes of points of X. If X is a Calabi-Yau n-fold, our construction can be considered as a higher dimensional generalization of Mukai's result.

Minimal presentation of the algebraic local cohomology via restriction of D-modules

Rémi Arcadias
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The aim of the talk is to give a minimal presentation of the algebraic local cohomology $O[1/f]/O$, considered as a module over the ring of linear partial differential operators D. Here O denotes the ring of convergent power series in n variables and f is a quasi homogeneous polynomial with an isolated singularity at the origin. The method employed is the D-module theoretic restriction. We will recall the restriction algorithm of Oaku and Takayama and explain how to use it in order to obtain a minimal presentation of the algebraic local cohomology.

A Pair of Quasi-Inverse Functors for an Extension of Perverse Sheaves

Delphine Dupont
Mathematical Institute of Oxford

In their article "Elementary construction of perverse sheaves", R. MacPherson and K. Vilonen show that on a Thom-Mather space X the category Perv$X$ of perverse sheaves is equivalent to the category $C(F, G, T)$ whose objects are data of perverse sheaves on the complementary of the closed strata S, a local system on S and some gluing data. To show this equivalence of categories, they define a functor C going from the category Perv$X$ to the category $C(F, G, T)$. This definition is based on the notion of perverse link. They do not define a quasi-inverse of this functor. Moreover they have to consider first the case where S is contractible and then they extend the equivalence to the topological case using the stack theory. In this talk we propose to consider what we call a perverse closed set, which is a bit different from a perverse link, in order to define a quasi-inverse to the functor C. Moreover we treat directly the topological case without using stack theory.

Laurent polynomials, hypergeometric systems and mirror symmetry

Thomas Reichelt
Institute of Mathematics, University of Mannheim

In this talk I will explain a close relationship between the Gauss-Manin systems of families of Laurent polynomials and the A-hypergeometric systems of Gelfand, Kapranov and Zelevinsky. As an application I will discuss the computation of the mirror Landau-Ginzburg model of a smooth, toric, Fano variety.

A survey on logarithmic D-modules

Luis Narváez Macarro
Universidad de Sevilla

We will lecture on some applications of D-module theory to the study of logarithmic de Rham complexes. We will review some of the main results obtained in the past 10-15 years, and we will describe some explicit interesting examples of D-modules which appear.

Cycle spaces of flag domains and D-modules

Corrado Marastoni
Universitá di Padova

Let G be a semisimple algebraic group, P a parabolic subgroup, $X = G/P$ the corresponding generalized flag manifold and G_0 a real form of G. A flag domain (i.e. a open G_0-orbit) D in X could have a complicated geometry; anyway, one can associate to it - by means of Matsuki correspondence - a “cycle space” $M(D)$ which is a Stein manifold. In this talk we introduce the above framework, and then we study, using the formalism of D-modules and homological algebra, a natural integral transform which transfers data from D to $M(D)$.

Characteristic classes of singular hypersurfaces

Laurentiu Maxim
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An old problem in geometry and topology is the computation of topological and analytical invariants of complex hypersurfaces, e.g., Betti numbers, Euler characteristic, signature, Hodge-Deligne numbers, etc. While the non-singular case is easier to deal with, the singular setting requires a subtle analysis of the intricate relation between the local and global topological and/or analytical structure of singularities. In this talk I will explain how to compute characteristic classes of complex hypersurfaces in terms of local invariants of singularities. This is joint work with Cappell, Shaneson, Saito and Schuermann.

Singularities and D-modules. The logarithmic de Rham complex

Francisco J. Calderón Moreno
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In this talk, we recall and refine some results in [2] and [3], and focus on the different properties associated with a divisor, as, for example, the (commutative or differential) linear type property for a free divisor. Previously, we recall some notions and basic results on Lie-Rinehart algebras, free divisors, the Bernstein construction and the Koszul property. We enunciate a general result (in [4]) based on D-modules theory and on our previous results in [1, 2, 3]:

Let X be a n-dimensional complex analytic manifold, $D \subset X$ a Koszul free divisor of commutative linear type (e.g. a locally quasi-homogeneous free divisor), $j : U = X\backslash D \hookrightarrow X$ the corresponding open inclusion, $\mathcal{E}$ an integrable logarithmic connection with respect to D and L the local system of the horizontal sections of $\mathcal{E}$ on U. Then, the canonical morphisms

$$\Omega^\bullet_X(\log D)(\mathcal{E}(kD)) \to Rj_*\mathcal{L},\ j_!\mathcal{L} \to \Omega^\bullet_X(\log D)(\mathcal{E}(-kD))$$

are locally isomorphisms in the derived category of sheaves of complex vector spaces for $k \gg 0$.

This result generalizes the logarithmic comparison theorem proved in [5] for $\mathcal{E} = \mathcal{O}_X$ and the case of normal crossing divisors [[6], II, §6].

The main purpose of the talk is not to explain the commented results, but give an insight into the possibilities of the D-module theory in order to classify or measure the singularities of a divisor.

References

[1] F. J. Calderón-Moreno. *Logarithmic differential operators and logarithmic de Rham complexes relative to a free divisor.* Ann. Sci. Ecole Norm. Sup. (4), 32(5) (1999), 701714. (math.AG/9807047).

[2] F. J. Calderón-Moreno and L. Narváez-Macarro. *The module Dfs for locally quasi- homogeneous free divisors.* Compositio Math., 134(1) (2002), 5974. (math.AG/0206262).

[3] F. J. Calderón Moreno and L. Narváez Macarro. *Dualité et comparaison sur les complexes de de Rham logarithmiques par rapport aux diviseurs libres.* Ann. Inst. Fourier (Grenoble), 55(1) (2005), 4775. (math.AG/0411045).

[4] F. J. Calder'on-Moreno and L. Narv'aez Macarro. *On the logarithmic comparison theorem for integrable logarithmic connections.* Proc. London Math. Soc. (3) 98(2009), 585-606. (math.AG/0603003).

[5] F. J. Castro-Jiménez, D. Mond, and L. Narváez-Macarro. *Cohomology of the complement of a free divisor.* Trans. Amer. Math. Soc., 348 (1996), 30373049.

[6] P. Deligne. *Equations Différentielles à Points Singuliers Réguliers.* Lect. Notes in Math. 163, Springer-Verlag, Berlin-Heidelberg, 1970.

Some combinatorial remarks on normal flatness in analytic spaces

M. J. Soto

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This is a joint work with José M. Tornero. In this article we present a combinatorial treatment of normal flatness in analytic ideals and rings, using the idea of equimultiple standard bases. We will prove, using purely combinatorial methods a characterization theorem for normal flatness. This will lead us to a new proof of a classical theorem on normal flatness, which can be stated by saying that normal flatness at a point along a smooth subspace is equivalent to the Hilbert function being locally constant.

Though these topics belong to classical analytic geometry, we believe that this approach is valuable, since it replaces extremely general algebraic theorems by the combinatorial object $u(I) \subset \mathbb{Z}_0^c$, obtaining new results and striking the combinatorial nature of the classical (and basic) ideas in the resolution of singularties.

Monodromy Zeta function formula for embedded Q-resolutions

Jorge Martín-Morales

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In a previous joint work with E. Artal Bartolo and J. Ortigas-Galindo, we have introduced the notion of embedded Q-resolution, which essentially consists in allowing the final ambient space to contain abelian quotient singularities, see [2, 3]. In this talk we will give a generalization of A'Campo's formula [1] for the monodromy zeta function of a singularity in this setting.

Its proof is based on a result by Dimca [4] and hence one needs to deal with constructible complexes of sheaves with respect to a stratification and the nearby cycles associated with an analytic function.

In particular, we prove that only the strata belonging to just one irreducible component of the natural stratification of the exceptional divisor contribute to $Z(f;t)$. This reflects the good behavior of abelian quotient singularities with respect to normal crossing divisors. By contrast, non-abelian groups seem to work differently.

This work is motivated by the fact that the combinatorial and computational complexity of embedded Q-resolutions is much simpler than the one of the classical embedded resolutions, but they keep as much information as needed for the comprehension of the topology of the singularity.

This problem has been considered in [5] for plane curve singularities.

References

[1] Norbert A'Campo. *La fonction zêta d'une monodromie.* Comment. Math. Helv., 50:233-248, 1975.

[2] E. Artal Bartolo, J. Martín-Morales, and J. Ortigas-Galindo. *Cartier and Weil divi- sors on varieties with quotient singularities.* ArXiv e-prints, April 2011.

[3] E. Artal, J. Martín-Morales, and J. Ortigas-Galindo. *Intersection theory on abelian- quotient V -surfaces and Q-resolutions.* ArXiv e-prints, May 2011.

[4] Alexandru Dimca. *Sheaves in topology.* Universitext. Springer-Verlag, Berlin, 2004.

[5] Willem Veys. *Zeta functions for curves and log canonical models.* Proc. London Math. Soc. (3), 74(2):360-378, 1997.

The subanalytic sites and the Riemann-Hilbert correspondences

Giovanni Morando

Universitá di Padova

In the first part of this talk we will recall the Riemann–Hilbert correspondences: the regular one, the local irregular one on curves and the results known regarding the general irregular case. We will try to explain through basic examples the main different steps corresponding to the formal classification of irregular ordinary linear differential equations and the study of their Stokes phenomena. In the second part of this talk, we will introduce the subanalytic site and the sheaf of tempered holomorphic functions. By means of easy examples we will explain their usefulness in the formal classification. We will point out the improvements achieved with these tools with respect to the classical ones. Further, we will discuss some results obtained on the constructibility of tempered solutions of holonomic D-modules which was conjectured in 2003 by M. Kashiwara and P. Schapira. If time allows, we will also show the link between the subanalytic site and spectral, Berkovich and Huber spaces.

Character theory for monodromy representations

Pietro Polesello

Universitá di Padova

Given a locally constant stack S on a topological space X, its character is a locally constant sheaf on the free loop space LX (in the case of $X = BG$ for a given complex Lie group G, it is a very simple Lusztig's character sheaf on G). This is defined by means of the categorical character, a notion introduced by several authors (Ganter-Kapranov, Bartlett-Willerton, Toen-Vezzosi, ecc.). In this talk I will first recall the character construction. Then I will show the behavior of the 2-monodromy representation of a locally constant stack and of its character sheaf under the direct image by a Serre fibration. (Joint work with Delphine Dupont)

Algebraic analysis and symplectic geometry

Stephane Guillermou
CNRS-UJF Grenoble

I will explain how methods of algebraic analysis, namely the microsupport of sheaves of Kashiwara and Schapira, can be used to recover non-displaceability results in symplectic geometry. The main problem considered in the talk is the existence of a sheaf whose microsupport is a given smooth Lagrangian submanifold of a cotangent bundle.

This is a joint work with M. Kashiwara and P. Schapira.

Higher Dimensional Rigid Local Systems II

Pedro Silva
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We construct rigid local systems on the complementary of an arbitrary plane curve with prescribed local data. We prove in this way the existence of a regular holonomic D-module and a special function associated to the local data refered above.

Extension of functors for algebras of formal deformation

Ana Rita Martins
Faculdade de Engenharia da Universidade Católica Portuguesa

Given two complex manifolds X and Y and a right exact functor between Serre subcategories $\mathcal{S}$ and $\mathcal{S}'$ of modules over an algebra of formal deformation in the sense of Kashiwara and Schapira, respectively on X and on Y, we give conditions for the existence of a canonical extension to the subcategory of modules such that the cohomology associated to the action of the formal parameter $\hbar$ belongs to $\mathcal{S}$. We give an explicit construction and prove that when the initial functor is exact so is its extension. We apply our construction to give a meaning to specialization along a submanifold in the framework of $\mathcal{D}[[\hbar]]$-modules. We also obtain a comparison theorem for regular holonomic $\mathcal{D}[[\hbar]]$-modules. This a joint work with Teresa Monteiro Fernandes and David Raimundo.

Limits of Tangents of a Quasi-Ordinary Hypersurface

António Araújo
Universidade Aberta

We compute the limits of tangents of a Quasi-Ordinary Hypersurface. We show that the limits of tangents of a QO hypersurface only depends on the topology of the hypersurface and its tangent cone. Hence the triviality of the limits of tangents is a topological invariant of a QO hypersurface. A germ of Lagrangean variety is in generic position at a point p if and only if the limit of tangents of its projection is trivial. If a D-module has characteristic variety in generic position at a point p, the limit of tangents at $\pi(p)$ of its singular locus is trivial. Joint work with Orlando Neto.

Functorial properties of $D[[\hbar]]$-modules

David Raimundo
CMAF, Universidade de Lisboa

We consider the ring $D[[\hbar]]$ of differential operators with a formal parameter $\hbar$ on a complex manifold, recently introduced by Kashiwara-Schapira, and we study wether or not the classical properties of D-modules extend to $D[[h]]$-modules. Namely, in this talk, we will present some results from a joint work with A.R. Martins and T. Monteiro Fernandes regarding the direct image and inverse image of $D[[\hbar]$-modules. We also see how to extend to this framework the results of Schapira-Schneiders on regularity, finiteness and duality of elliptic pairs.

Resolution of Singularities of Quasi-Ordinary Legendrian Surfaces

João Cabral
CMAF, Universidade Nova de Lisboa

We show that the canonical algorithm of resolution of singularities produces the desingularization of the conormal of a quasi-ordinary surface singularity. The computation of "logarithmic" limits of tangents of a quasi-ordinary surface is a essential tool of the proof. This theorem implies a desingularization theorem for regular holonomic D-modules. This is a joint work with António Araújo and Orlando Neto.

On the Laplace transform for temperate holomorphic functions

Andrea D'Agnolo
Universitá di Padova

In order to discuss the Fourier-Sato transform of not necessarily conic sheaves, we compensate the lack of homogeneity by adding an extra variable. We can then adapt a theorem by Kashiwara and Schapira on the Laplace transform for temperate holomorphic functions to obtain Paley-Wiener type results. As a key ingredient for this approach, we introduce the subanalytic sheaf of functions with exponential growth and relate it to the sheaf of tempered functions with an extra variable.