SOME BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Robert Hakl, Alexander Lomtatidze, Jiří Šremr

Mathematical Institute AS CR, Žižkova 22, 616 62 Brno, Czech Republic,
e-mail: `hakl@ipm.cz`, `bacho@math.muni.cz`, `sremr@ipm.cz`

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On the segment $[a, b]$, we consider the boundary value problem

$$u'(t) = F(u)(t), \tag{1}$$

$$\lambda u(a) + \mu u(b) = h(u), \tag{2}$$

where $F : C([a, b]; R) \to L([a, b]; R)$ is a continuous operator satisfying Carathèodory condition, $h : C([a, b]; R) \to R$ is a continuous functional, and $\lambda, \mu \in R$, $|\lambda|+|\mu| \neq 0$.

The important special case of the equation (1) is a differential equation with a deviating argument of the form

$$u'(t) = f(t, u(t), u(\tau(t))), \tag{3}$$

where $f : [a, b] \times R^2 \to R$ is a function satisfying Carathèodory conditions, and $\tau : [a, b] \to [a, b]$ is a measurable function.

The special cases of boundary condition (2) are, in particular, periodic condition ($\lambda = -\mu$), antiperiodic condition ($\lambda = \mu$), and the initial conditions ($\lambda = 0$, resp. $\mu = 0$).

The aim of the talk is to present the main results on the solvability and unique solvability of the problem (1), (2), resp. (3), (2), contained in the monography

> R. Hakl, A. Lomtatidze, J. Šremr, *Some Boundary Value Problems For First Order Scalar Functional Differential Equations*

which was published in the series “Folia” (Masaryk University Brno).

More detailed, there will be presented new optimal (unimprovable) sufficient conditions for solvability of problem (1), (2) both in the cases where F is linear and nonlinear operator. The question on the sign of the solution is discussed, as well.