Spike bound states for Schrödinger equations

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8th September 2005

Abstract

We consider a class of stationary nonlinear Schrödinger equations of the form

$$\begin{cases} -\varepsilon^2 \Delta u + V(x)u = K(x)u^p, \ x \in \Omega \\ u \in H_0^1(\Omega), \ u > 0, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $1 < p < (N+2)/(N-2)$ if $N \geq 3$, V and K are positive functions. It is known that, when V is bounded away from zero, such an equation exhibits spike solutions, i.e. solutions that concentrate around a finite number of points $x_i \in \Omega$, as $\varepsilon \to 0$. In a recent paper, Ambrosetti, Felli and Malchiodi consider, in $\Omega = \mathbb{R}^N$, the case of a potential $V(x) \sim |x|^{-\alpha}$ as $|x| \to \infty$, with $0 < \alpha < 2$. Assuming that $K(x) \sim |x|^{-\beta}$ as $|x| \to \infty$ with $\beta > 0$ and a compatibility condition between p, α and β, they prove the existence of a ground state that concentrates at a global minimum point of the auxiliary function

$$A(x) = V(x)^{\frac{p+1}{p-1} - \frac{N}{2}} K(x)^{-\frac{2}{p-1}}.$$

In this talk, we consider situations where $0 = \inf_\Omega A$ is achieved as $|x| \to \infty$ and ground states do not exist. We therefore look for another type of solutions, namely we prove the existence of bound states that concentrate at local minimum points of A. Our approach is variational and rely on a suitable adaptation of the penalization method introduced by Del Pino and Felmer. To emphasize the main ideas and to avoid too much technicalities, we will focus in the talk on the one dimensional model

$$\begin{cases} -\varepsilon^2 u'' + V(x)u = K(x)u^p, \ x \in \mathbb{R} \\ u \in H^1(\mathbb{R}), \ u > 0. \end{cases}$$

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