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COLÓQUIO DE MATEMÁTICA

Monotone Operators as Convex Objects

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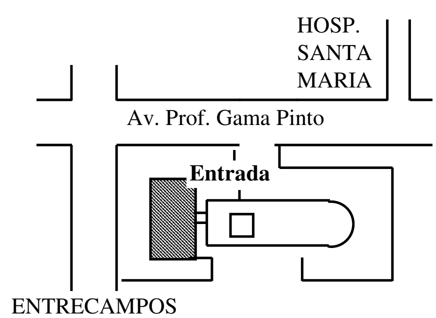

Monotone Operators as Convex Objects

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Dedication. In his famous '23 Mathematical Problems' talk of 1900 David Hilbert wrote

"Besides it is an error to believe that rigor in the proof is the enemy of simplicity."

This talk is dedicated to the memory of my collaborator and friend Simon Fitzpatrick (1953–2004) whose work was typified equally by rigour and simplicity.

Abstract. We say a multifunction $T : X \mapsto 2^{X^*}$ is *monotone* provided that for any $x, y \in X$, and $x^* \in T(x), y^* \in T(y)$,

$$\langle y - x, y^* - x^* \rangle \geq 0,$$

and that T is *maximal monotone* if its graph is not properly included in any other monotone graph. The *convex subdifferential* in Banach space and a *skew linear matrix* are the canonical examples of maximal monotone multifunctions. Maximal monotone operators play an important role in functional analysis, optimization and partial differential equation theory, with applications in subjects such as mathematical economics and robust control.

In this talk, based on [1], I shall show how—based largely on a long-neglected observation of Fitzpatrick [5]—the originally quite complex theory of monotone operators [6,7,8] can be almost entirely reduced to convex analysis, [2,3,4]. I shall also highlight various long standing open questions which these new techniques offer new access to.

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