The role of global bifurcations in ecosystem dynamics

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ABSTRACT

To study the long-term dynamical behaviour predicted by mathematical models of ecosystems, bifurcation analysis can be used. In addition to equilibria, limit cycles or chaotic behaviour, or combinations thereof, can occur. In such an analysis the parameter space is divided into regions where the system behaves qualitatively the same. The boundaries of these regions are formed by bifurcation points in which the system is structurally unstable. These points can be found by a classical local stability analysis. The procedure is to calculate the equilibrium or limit cycle, linearise, study stability of that equilibrium or limit cycle, and use the fact that the nonlinear system behaves the same as the approximating linear system in the neighbourhood of the equilibrium or limit cycle.

Contrary to these local bifurcations, with global bifurcations a local analysis is not sufficient. Saddle equilibrium points and limit cycles can be connected. Starting in the unstable manifold of one point or cycle integration in time gives an orbit through the state-space that ends via a stable manifold at the same point or cycle (homoclinic) or at the stable manifold of another point or cycle (heteroclinic).

We will present numerical bifurcation analysis results for a number of previously published food chain models focusing on global bifurcations. One is the well-know Rosenzweig-MacArthur model [1] which has been studied intensively. We will link the occurrence of the Shil'nikov homoclinic point-to-point, heteroclinic point-to-cycle and the homoclinic cycle-to-cycle global bifurcation. The Shil'nikov bifurcation forms a skeleton for a cascade of flip and tangent bifurcations and is associated chaotic dynamics. The homoclinic cycle-to-cycle global bifurcation is related to crises where, under parameter variation, the chaotic attractor suddenly appears or disappears.

The results were obtained using methods described in [2,3] for the numerical continuation of point-to-cycle and cycle-to-cycle connecting orbits in 3-dimensional autonomous ODE's. Projection boundary conditions are used in the BVP formulation allowing a straightforward implementation in AUTO [4], in which only the standard features of the software are employed. Complete AUTO demos, which can be easily adapted to any autonomous 3-dimensional ODE system, are available: `http://www.bio.vu.nl/thb/research/project/globif/` and AUTO2007p version 0.6 `http://indy.cs.concordia.ca/auto/`.

Furthermore the analysis of two models published in literature are revisited. In [5] also a three trophic food chain is analysed. The results presented in that paper will be put into context by comparison with results of a more detailed analysis taking global bifurcation into account. In that model coexisting chaotic attractors occur. Here the homoclinic cycle-to-cycle global bifurcation is also related to crises where the chaotic attractor changes shape or merge.

In [6] the first two trophic levels of a food chain are modelled where both producer and grazer are composed of two essential elements: carbon and phosphorous. The Liebig minimum law is used to model the trophic interaction. Then system-parameters (e.g. growth rate) depend discontinuously on the state variables. As a consequence the Jacobian matrix

evaluated at an equilibrium is discontinuous with respect to a bifurcation parameter. Bifurcation points different from those occurring in the classical smooth models are found. These new bifurcation points are analysed numerically. Moreover the results will be compared with those of a smooth alternative model where all system-parameters depend smoothly on the state variables.

Key Words: Boundary crises, boundary value problems, chaos, global bifurcations, homotopy, projection boundary conditions

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