CENTRO DE MATEMÁTICA E APLICAÇÕES FUNDAMENTAIS

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Mês de: NOVEMBRO 2014

SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

(Alteração da Hora do Seminário)

Dia 6 de Novembro (quinta-feira), às 13:40h, na Sala B3-01

A variational characterization of critical speed for travelling waves in nonlinear diffusion

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Abstract:

When we look for travelling waves with a monotone profile to the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right] + f(u), \tag{1}$$

where $f : [0,1] \to \mathbb{R}$ is a continuous function such that $f(0) = f(1) = 0$ and $f(u) > 0$ if $u \in (0,1)$, we are faced with finding positive solutions to the second order boundary value problem

$$(|u'|^{p-2}u')' + cu' + f(u) = 0 \tag{2}$$

$$u(-\infty) = 1, \;\; u(+\infty) = 0. \tag{3}$$

where $c > 0$ is a parameter (the speed of the wave). It has been shown that, as in the classical FKPP theory, there exists a minimal speed $c^* > 0$. Now let q be the conjugate exponent,

$$\frac{1}{p} + \frac{1}{q} = 1.$$

We relate the problem (2)-(3) with another singular boundary value problem: find solutions $0 \le v < 1$ of

$$(v'^{q-1})' + \lambda\frac{f(v(s))}{s^q} = 0 \tag{s}$$

(λ is a positive parameter). We conclude that there is a relationship between c^*, (s) and the constrained minimum problem

$$\gamma = \inf\{\int_0^\infty \frac{v'(s)^q}{q}\,ds \mid \int_0^\infty \frac{F(v(s))}{s^q}\,ds = 1\}$$

where $F(x) = \int_0^x f(z)\,dz$ and f is extended to $\mathbb{R}$ with value 0 outside $[0,1]$. In fact

$$\gamma = \frac{q}{p}\frac{1}{c^{*q}}$$

and we give conditions under which the infimum is attained.

This is joint research with Andrea Gavioli (Modena).

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