UNIVERSIDADE DE LISBOA
CENTRO DE MATEMÁTICA E APLICAÇÕES FUNDAMENTAIS
Av. Prof. Gama Pinto 2, 1649-003 LISBOA, PORTUGAL Tel. (351) 217 904 700 FAX (351) 217 954 288

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SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

Dia 11 de Julho (quinta-feira), às 13h30, na Sala B3-01

On the Leray-Hopf condition: the restricted flux condition, and the inflow case

Hugo Beirão da Veiga

(Universidade de Pisa)

Abstract:

We consider the stationary Navier-Stokes equations in a bounded, regular, domain Ω for inhomogeneous boundary data. We assume the necessary compatibility condition which requires that the total flux across the boundary is zero (a consequence of the incompressibility). If the boundary has only one connected component, this condition is also sufficient to the existence of the solution. In the general case the problem is still open. A classical sufficient condition is the so called Leray-Hopf inequality, to be satisfied by suitable solenoidal extensions of the boundary data. If the net flux of the given boundary data vanishes across each single component of the boundary (necessarily true under one component) it was proved (Leray, 1933, Hopf 1941) that such a solenoidal extension exists. The proof will be outlined in the first part of the talk.

In the second part, following the notation used by Prof. Bemelmans in the next talk, I consider a two dimensional domain $\Omega = A \setminus \overline{B},\ B \subset\subset A,\ A, B$ simply connected, that is topologically equivalent to an annulus. A. Takeshita, *A remark on Leray's inequality,* Pac. J. Math. 157 (1993), 151-158, showed that there is no extension that fulfills the desired inequality if the flux through each boundary component differs from zero, and if there are $B_r(0)$ and $B_R(0)$ with $B \subset\subset B_r(0) \subset\subset B_R(0) \subset\subset A$. Actually, Takeshita only considers the case in which the flow into Ω through ∂B is negative as is the flow out of Ω through ∂A (the inflow case). We give an outline of the proof in this case. The more involved outflow case will be treated by Prof. Bemelmans.

It is worth noting that in a recent paper (in spite of the negative reply to the Leray-Hopf condition), V. Korobkov, K. Pileckas, and R. Russo show that the original N.-S. problem, in the above two dimensional case and for two boundary components, do have solution, at least in the inflow case.

Local:
Instituto para a Investigação Interdisciplinar
da Universidade de Lisboa
Av. Prof. Gama Pinto, 2
1649-003 Lisboa