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SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

Dia 28 de Fevereiro (quinta-feira), às 13h30, na Sala B3-01

On the nonlocal *p*-Laplacian equation and some of its applications

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Abstract:

This lecture deals with the nonlocal $p-$Laplacian type diffusion equation,

$$u_t(t,x) = \int_\Omega J(x-y)|u(t,y)-u(t,x)|^{p-2}(u(t,y)-u(t,x))\,dy.$$

If $p > 1$, this is the nonlocal analogous problem to the well known local $p-$Laplacian evolution equation $u_t = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ with homogeneous Neumann boundary conditions. We prove existence and uniqueness of a strong solution, and if the kernel J is rescaled in an appropriate way, we show that the solutions to the corresponding nonlocal problems converge strongly in $L^\infty(0,T;L^p(\Omega))$ to the solution of the $p-$laplacian with homogeneous Neumann boundary conditions. The extreme case $p = 1$, that is, the nonlocal analogous to the total variation flow, is also analyzed.

We also study the nonlocal $\infty-$Laplacian type diffusion equation, obtained as the limit as $p \to \infty$ of solutions to the nonlocal analogous to the $p-$Laplacian evolution,

$$u_t(t,x) = \int_{\mathbb{R}^N} J(x-y)|u(t,y)-u(t,x)|^{p-2}(u(t,y)-u(t,x))\,dy.$$

We prove existence and uniqueness of a limit solution that verifies an equation governed by the subdifferetial of a convex energy functional associated to the indicator function of the set $K = \{u : |u(x)-u(y)| \leq 1, \text{ when } x-y \in supp(J)\}$. We also find some explicit examples of solutions to the limit equation.

If the kernel J is rescaled in an appropriate way, we show that the solutions to the corresponding nonlocal problems converge strongly in $L^\infty(0,T;L^2(\Omega))$ to the limit solution of the local evolutions of the $p-$laplacian, $v_t = \Delta_p v$. This last limit problem has been proposed as a model to describe the formation of a sandpile.

Finally, we give an interpretation of the limit problem in terms of Monge-Kantorovich mass transport theory.

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