1 One-Phase Parabolic Free Boundary Problem in Convex Domain

Abstract. Let us be given a domain $\Omega_0 \subset \mathbb{R}^n \times [0, \infty)$ with convex time sections for which the set $K_0 := \Omega_0 \cap \{t = 0\}$ is not empty, and a continuous function $u_0(x), x \in \mathbb{R}^n \setminus K_0$ for which the set $K_1 := \text{supp} u_0 \cup K_0$ is convex. We are looking for a pair $(u, \Omega_1), \Omega_0 \subset \Omega_1 \subset \mathbb{R}^n \times [0, \infty)$, $u \in C^2(\Omega_1 \setminus \overline{\Omega}_0) \cap C(\overline{\Omega}_1 \setminus \Omega_0)$ which is the solution for the following problem:

$$\begin{cases} u_t = \Delta u & \text{in } \Omega_1 \setminus \overline{\Omega}_0 \\ u(x,t) = 1 & \text{on } \Gamma_0 \\ u(x,t) = 0 & \text{on } \Gamma_1 \\ |Du(x,t)| = 1 & \text{on } \Gamma_1 \\ u(x,0) = u_0(x) & \text{in } K_1 \setminus \overline{K}_0 \end{cases}$$

where Γ_i is the lateral boundary of Ω_i, $i = 0, 1$. We show, that this problem has unique solution.