Mathematical analysis of the discharge of a laminar hot gas in a colder atmosphere

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We consider a boundary layer approximation in the problem of the discharge of a laminar hot gas in a stagnant colder atmosphere of the same gas [2],[3]. This approximation leads to the nonlinear system of partial differential equations

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial r}(\rho v) = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{\partial}{\partial r}\left(\mu \frac{\partial u}{\partial r}\right) + G\left(1 - \frac{\varepsilon}{T}\right),\ \rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial r} = \frac{1}{Pr}\frac{\partial}{\partial r}\left(\mu \frac{\partial T}{\partial r}\right), \tag{2}$$

where Pr is the Prandtl number , G is the Froude number (are given positive numbers). The system is completed with the constitutive conditions $\rho = 1/T,\ \mu = T^{\sigma}$(for some $0 < \sigma < \infty$). Here the unknowns are the vector velocity (v, u), and the temperature T. System (1)-(2) is considered in the domain $\Omega = \{(x, r) \in \mathbf{R}^2 : 0 < x < \infty, 0 < r < l \leq \infty\}$ with the boundary conditions

$$\frac{\partial u}{\partial r} = v = \frac{\partial T}{\partial r} = 0, \quad \text{for } r = 0, \quad \text{and for } x > 0, \tag{3}$$

$$u = \delta,\ T = \varepsilon,\ \text{for } r = l, \quad \text{and for } x > 0, \tag{4}$$

$$u(0, r) = u_0(r) \geq \delta,\ T(0, r) = T_0(r) \geq \varepsilon \text{ for } x = 0 \text{ and for } r \in [0, l]. \tag{5}$$

Notice that, although arising in stationary regime, the system is of parabolic type and that condition (5) looks as an initial condition if we understood variable x as the "fictitious" time.

We prove existence and uniqueness of solutions of the nondegenerate problem (corresponding to the assumption $\delta > 0$ and $\varepsilon > 0$). We also study the limit case ($\delta = 0$ and $\varepsilon = 0$) leading to the degeneracy of the system for which we prove the generation of some interfaces given as the boundaries of the support of (u, T). Using the method of local energy estimate [1], we show that the solution possesses the localization properties such that- finite speed of propagation, waiting time property, extinction with respect to x. We discuss also some results of numerical calculations for this problem. To prove the existence and uniqueness of solutions we use the, so called, von Mises variables (x, ψ), (where ψ is the associated "stream function") which transform (1)-(2) into the purely diffusive system (eliminating the unknown v)

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial \psi}\left(T^{\sigma-1} u \frac{\partial u}{\partial \psi}\right) + \frac{TG}{u}\left(1 - \frac{\varepsilon}{T}\right), \frac{\partial T}{\partial x} = \frac{1}{Pr}\frac{\partial}{\partial \psi}\left(T^{\sigma-1} u \frac{\partial T}{\partial \psi}\right).$$

References

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