Euler equations with non-homogeneous Navier slip boundary condition

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We consider the motion of an ideal fluid in a 2D-bounded domain, admitting flows through the boundary of this domain.

The motion of the fluid in a domain $\Omega \subseteq \mathbb{R}^2$ is described by the Euler equations

$$\mathbf{v}_t + \operatorname{div}(\mathbf{v} \otimes \mathbf{v}) - \bigtriangledown p = 0, \qquad (\mathbf{x}, t) \in \Omega_T := \Omega \times (0, T), \tag{1}$$

$$\operatorname{div} \mathbf{v} = 0, \qquad (\mathbf{x}, t) \in \Omega_T \tag{2}$$

with a given initial condition

$$\mathbf{v}(\mathbf{x}, 0) = \mathbf{v}_0(\mathbf{x}), \qquad \mathbf{x} \in \Omega \tag{3}$$

and *non-homogeneous* Navier slip boundary conditions on the boundary of the domain Ω:

$$\mathbf{v} \cdot \mathsf{n} = a, \qquad \mathbf{x} \in \Gamma_T := \Gamma \times (0, T), \tag{4}$$

$$2D(\mathbf{v})\mathsf{n} \cdot \mathsf{s} + \alpha \mathbf{v} \cdot \mathsf{s} = b, \qquad \mathbf{x} \in \Gamma_T^- := \Gamma^- \times (0, T). \tag{5}$$

Here $\mathbf{v}(\mathbf{x}, t)$ is the velocity of the fluid; $p(\mathbf{x}, t)$ is the pressure; the tensor $D(\mathbf{v})$ is the rate of strain of the velocity $\mathbf{v}$; (n, s) is the pair formed by the outside normal and tangent vectors to the boundary Γ of Ω; Γ^- is the part of Γ, where $\mathbf{v}\,\mathsf{n} = a < 0$.

The results:

1) We establish the solvability of this problem (1)-(5) realizing the passage to the limit in the Navier-Stokes equations with vanishing viscosity;

2) The solvability is proved in the class of weak solutions with L_p-bounded vorticity, $p \in (2, \infty]$;

3) It is shown that the weak solution satisfies the Navier slip boundary conditions (4)-(5).