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Mês de: Julho 2007

SEMINÁRIOS DE ANÁLISE

Dia 12 de Julho (quinta-feira), às **14h15**, na Sala B3-01

A bridge between Ordinary Differential Equations and One-Dimensional Maps via Delay Differential Equations

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Abstract:

Let us consider the family of delay differential equations

$$x'(t) = -x(t) + f(x(t-r)), \qquad (1)$$

where $r > 0$ is the delay, and f: IR→IR satisfies the negative feed-back condition $x\,f(x) < 0$ for $x \neq 0$. This equation appears as a model in many fields as biology, physiology, and economics. Obviously, in the limit case $r = 0$, Eq. (1) is an ordinary differential equation with very simple dynamics: all solutions converge to zero. Under additional conditions, the same dynamics is inherited by equation (1) when r is small enough.

Now, after a change of variables, one can see that the limit case of (1) as $r \to \infty$ gives the difference equation with continuous argument

$$x(t) = f(x(t-1)), \qquad (2)$$

whose dynamics is determined by the behaviour of the one-dimensional discrete dynamical system

$$x_n + 1 = f(x_n),\ n = 0,1,\ldots \qquad (3)$$

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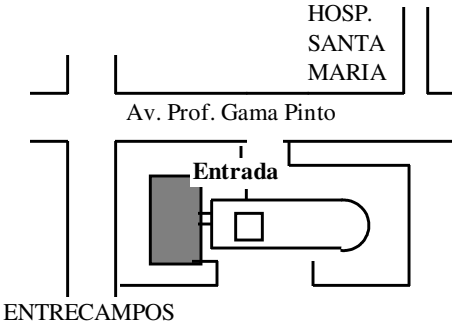

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In this way, when we consider the delay r as a parameter ranging from $r = 0$ to $r = +\infty$, we can observe the transition between the ODE $x'(t) = -x(t)+f(x(t))$ and the difference equation (2), obtained by substitution $x'(t) \approx x(t+1)-x(t)$.

In this talk, we analyze this transition from $r = 0$ to $r = \infty$ for a family of nonlinearities f, revisiting in this way some interesting results obtained in the literature for Eq. (1). Some related conjectures and open problems will be formulated.

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