Gravity Waves in Two-Layer Flows with Free Surface

Ricardo Barros* Sergey Gavrilyuk†

Abstract

In this work we study the wave propagation in two-layer flows with free surface. Two distinct classes of models are contemplated. First, we consider the "two-layer" version of the shallow water equations (also known by Saint-Venant's equations). This model is strictly hyperbolic for small relative velocities. It would be natural to consider this model as suitable for the description of hydraulic jumps. However, like most of models describing multi-velocity flows, the system is not presented in conservative form. We present a survey on the number of conservation laws available for the multi-dimensional case that seems to imply that the system is truly nonconservative. Therefore, the impossibility of presenting a complete set of Rankine-Hugoniot conditions enabling the characterization of weak solutions in the classical way.

Then, we obtain a dispersive model suited to the description of large amplitude waves propagating in the same physical system. The model is a "two-layer" generalization of the Green-Naghdi model and can be derived by applying Hamilton's principle to a Lagrangian that results from the insertion of approximations directly into the Lagrangian for the full water-wave problem. As a consequence, the variational structure of the original problem and the corresponding symmetry properties are preserved. In addition, it is a fully nonlinear model and deals with rotational flows. As in the case of the full problem, the present model captures the resonance between short waves and long waves. In this framework it is shown, by using numerical computations, the existence of homoclinic trajectories embedded into the continuous spectrum. These correspond to true solitary waves having the same velocities at infinity in each layer. Their study reduces to the analysis of a Hamiltonian system with two degrees of freedom. The traveling-wave solutions depend on three parameters: the *density ratio*, the *depth ratio* and the *Froude number* based on the bottom layer. Two wave regimes, characterized by the elevation or depression of the interface between the layers are presented. A critical depth ratio separates these two regimes and it will be shown how it relates to a change of the structure of the potential for the Hamiltonian system. The analysis of the number and nature of critical points turned out to be decisive in this work. It was found that the number of critical points can be four or two, depending on the value of the Froude number (for fixed density and depth ratios). For sets of parameters corresponding to oceanic conditions we have perceived the existence of true solitary waves and their broadening whenever the wave speed increases towards a limit value. Finally, other sets of parameters are considered for which multi-humped solitons exist, highlighting the richness and complexity of the system considered.

Keywords: Two-layer flow, conservation laws, Frobenius' problem, shallow water equations, Green-Naghdi model, dispersive nonlinear waves, internal waves, embedded solitary waves, Hamiltonian system.

*Université Aix-Marseille III and CNRS UMR 6595, IUSTI, 5 rue E. Fermi, 13453 Marseille Cedex 13, France, e-mail: ricardo.barros@polytech.univ-mrs.fr

†Université Aix-Marseille III and CNRS UMR 6595, IUSTI, 5 rue E. Fermi, 13453 Marseille Cedex 13, France, e-mail: sergey.gavrilyuk@polytech.univ-mrs.fr