Cycles and universality in sunspot numbers fluctuations

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ABSTRACT

We analyze the famous Wolf's sunspot numbers. Surprisingly, we discovered that the distribution of the sunspot number fluctuations for, both, the ascending and descending phase is close to the universal non-parametric Bramwell-Holdsworth-Pinton (BHP) distribution. Since the BHP probability density function appears in several other physical phenomena, our result reveals an universal feature of the Wolf's sunspot numbers.

Subject headings: Sun: sunspots — Physical data and processes: (magnetohydrodynamics:) MHD — methods and techniques: data analysis

1. Introduction

A sunspot is a region on the Sun's surface (photosphere) that is marked by a lower temperature than its surroundings and has intense magnetic activity. In 1848, the Swiss astronomer Johann Rudolph Wolf introduced a daily measurement of sunspot number. His method, which is still used today, counts the total number of spots visible on the face of the sun and the number of groups into which they cluster. The actual sunspot numbers consists of 23 cycles [1]. It was Samuel Schwabe (14) whom for the first time suggested a probable period of ten years (i.e. that at every tenth year the number of spots reached a maximum). The average duration of the sunspot cycle (also called Schwabe cycle) is 133 months (11.08 years), but cycles as short as 9 years and as long as 14 years have been observed, (see Rabin et al. (13)). Usually, the change from cycle to cycle consists of 1-1.5 years time when there are spots from both old and new cycles. The traditionally used breakpoint is when there is a takeover, and the decision of when this takeover occurs is taken by an international conference of astronomers.

Here, we analyze the monthly sunspot numbers that is a time series of about 3000 values corresponding to the monthly sums of sunspot numbers. Our start point corresponds to the sunspot minimum of the first cycle (March of 1755) fully observed. We have chosen the first observation to be the first sunspot minimum. Since the descending and ascending phases have different characters, we analyze separately the two phases. Surprisingly, we observe, in this paper, that the Wolf's sunspot numbers fluctuates according to the universal non-parametric BHP distribution for, both, the ascending and descending phases. In particular, the histograms of, both, the ascending and descending fluctuations variables do not follow a gaussian distribution. Both distributions exhibit heavy tails and a universal non-zero skewness. For any given time of the ascending or descending phase, our result, also, gives an estimator for the probability of any given measurable set of sunspot numbers.

2. Universality of the Bramwell-Hodsworth-Pinton distribution

The universal nonparametric BHP pdf was discovered by Bramwell, Holdsworth and Pinton (3). The universal nonparametric BHP pdf is the pdf of the fluctuations of the total magnetization, in the strong coupling (low temperature) regime for a two-dimensional spin model (2dXY), using the spin wave approximation. The magnetization distribution, that they found, is named, after them, the Bramwell-Holdsworth-Pinton (BHP) distribution. The *BHP probability density function (pdf)* is given by

$$p(\mu) = \int_{-\infty}^{\infty} \frac{dx}{2\pi} \sqrt{\frac{1}{2N^2} \sum_{k=1}^{N-1} \frac{1}{\lambda_k^2}} \quad e^{ix\mu\sqrt{\frac{1}{2N^2}\sum_{k=1}^{N-1}\frac{1}{\lambda_k^2}} - \sum_{k=1}^{N-1}\left[\frac{ix}{2N}\frac{1}{\lambda_k} - \frac{i}{2}\arctan\left(\frac{x}{N\lambda_k}\right)\right]}$$

$$.e^{-\sum_{k=1}^{N-1}\left[\frac{1}{4}\ln\left(1+\frac{x^2}{N^2\lambda_k^2}\right)\right]} \quad , \tag{1}$$

[1]The data and related information on the sunspot numbers is available at the Solar Data Services site, http://www.ngdc.noaa.gov/stp/SOLAR/SSN/ssn.html. of the *National Geophysical Data Center*

where the $\{\lambda_k\}_{k=1}^{L}$ are the eigenvalues, as determined in (5), of the adjacency matrix. It follows, from the formula of the BHP pdf, that the asymptotic values for large deviations, below and above the mean, are exponential and double exponential, respectively (in this article, we use the approximation of the BHP pdf obtained by taking $L = 10$ and $N = L^2$ in equation (1)). As we can see, the BHP distribution does not have any parameter (except the mean that is normalize to 0 and the standard deviation that is normalized to 1) and it is universal, in the sense that appears in several physical phenomena. For instance, the universal nonparametric BHP distribution is a good model to explain the fluctuations of order parameters in theoretical examples such as, models of self-organized criticality, equilibrium critical behavior, percolation phenomena (see Bramwell et al. (3)), the Sneppen model (see Bramwell et al. (3) and Dahlstedt and Jensen (7)), and auto-ignition fire models (see Sinha-Ray et al. (15)). The universal nonparametric BHP distribution is, also, an explanatory model for fluctuations of several phenomenon such as, width power in steady state systems (see Bramwell et al. (3)), fluctuations in river heights and flow (see Bramwell et al. (5) and Dahlstedt and Jensen (8)) and for the plasma density fluctuations and electrostatic turbulent fluxes measured at the scrape-off layer of the Alcator C-mod Tokamaks (see Van Milligen et al. (16)). Surprisingly, we observe that the Wolf's sunspot numbers fluctuates according to the universal nonparametric BHP distribution for, both, the ascending and descending phase. Hence, our result reveals an universal feature of the Wolf's sunspot numbers.

3. Wolf's sunspot numbers

The sunspot numbers $\{X_t\}$ consists, actually, of 23 cycles, whose last one has still not ended. Since the ascending and descending phases have different characters, we pass to analyze separately the two phases.

Let M_k be the *month corresponding to the maximum value* X_{M_k} of the monthly sunspot numbers k-th cycle for $k \in \{1 \ldots 23\}$. Let m_k be the *month corresponding to the minimum value* X_{m_k} of the monthly sunspot numbers k-th cycle.

3.1. Ascending phase

The *duration* a_k of the ascending phase of the k-th sunspot cycle is given by $a_k = m_k - M_k$ (see figure 1). The *k-th ascending phase variable* A_t^k is defined by

$$A_t^k = X_{t+m_k},$$

where $t \in \{0, \ldots, a_k\}$ (see figure 3). Let $\mathcal{A}(t)$ denote the set of all k's such that the ascending phase A_t^k has durations a_k higher than t, i.e.

$$\mathcal{A}(t) = \{k : t \leq a_k\} \quad .$$

Let $\mathcal{T}^a$ be the *minimum* t subjected to $\#\mathcal{A}(t) > 1$, i.e.

$$\mathcal{T}^a = \max\{t : \#\mathcal{A}(t) > 1\} \quad .$$

Hence, there are at least two ascending phases t months long, for every $t \leq \mathcal{T}^a$. We define the *ascending mean* μ_t^a by

$$\mu_t^a = \frac{1}{\#\mathcal{A}(t)} \sum_{k \in \mathcal{A}(t)} A_t^k,$$

where $t \in \{0, \ldots, \mathcal{T}^a\}$ (see figure 3). We define the *ascending standard deviation* σ_t^a by

$$\sigma_t^a = \sqrt{\frac{1}{\#\mathcal{A}(t)} \sum_{k \in \mathcal{A}(t)} (A_t^k - \mu_t^a)^2},$$

where $t \in \{0, \ldots, \mathcal{T}^a\}$ (see figure 3). For each $t \in \mathcal{T}^a$, we define the *ascending fluctuations variables* $A_{t,k}^f$ by

$$A_{t,k}^f = \frac{A_t^k - \mu_t^a}{\sigma_t^a},$$

where $k \in \{1 \ldots, 23\}$ and $t \in \{0 \ldots, \mathcal{T}^a\}$. The ascending fluctuations variables $A_{t,k}^f$ measures the deviations of the sunspot ascending phases A_t^k to the ascending mean μ_t in standard deviation σ_t^a units. Surprisingly, the histogram of the aggregated ascending observed fluctuations shows a data collapse to the universal nonparametric BHP pdf (see figure 5). In particular, the histogram of the ascending fluctuations variables $A_{t,k}^f$ do not follow a gaussian distribution, exhibiting heavy tails and a universal non-zero skewness. The highest observed positive fluctuation $A_{t,k}^f$ is equal to 3.604 and the lowest observed negative fluctuation $A_{t,k}^f$ is -1.894 showing the asymmetries of the histogram. We get an estimator for the sunspot numbers

$$A_t^k = \sigma_t^a A_{t,k}^f + \mu_t^a, \tag{2}$$

using the ascending mean μ_t^a and the ascending standard deviation σ_t^a, as obtained in figure 3, and noting that $A_{t,k}^f$ follows the universal nonparametric BHP pdf. In figure 3, we observe that the highest ascending means μ_t^a occur together with the highest standard deviations σ_t^a, for values of t close to 44. Hence, by equation (2), the highest sunspot numbers A_t^k occur for values of t close to 44.

3.2. Descending phase

The *duration* d_k of the descending phase of the k-th sunspot cycle is given by $d_k = M_k - m_{k+1}$ (see figure 2). The *k-th descending phase variable* D_t^k is defined by

$$D_t^k = X_{t+M_k},$$

where $t \in \{0, \ldots, d_k\}$ (see figure 4). Let $\mathcal{D}(t)$ denote the set of all k's such that the descending phase D_t^k has durations d_k higher that t, i.e.

$$\mathcal{D}(t) = \{k : t \leq d_k\} \quad .$$

Let $\mathcal{T}^d$ be the *minimum* t subjected to $\#\mathcal{D}(t) > 1$, i.e.

$$\mathcal{T}^d = \max\{t : \#\mathcal{D}(t) > 1\} \quad .$$

Hence, there are at least two descending phases t months long, for every $t \leq \mathcal{T}^d$. We define the *descending mean* μ_t^d by

$$\mu_t^d = \frac{1}{\#\mathcal{D}(t)} \sum_{k \in \mathcal{D}(t)} D_t^k$$

where $t \in \{0, \ldots, \mathcal{T}^d\}$ (see figure 4). We define the *descending standard deviation*, σ_t^d by

$$\sigma_t^d = \sqrt{\frac{1}{\#\mathcal{D}(t)} \sum_{k \in \mathcal{D}(t)} (D_t^k - \mu_t^d)^2}$$

where $k \in \{1 \ldots, 23\}$ and $t \in \{0, \ldots, \mathcal{T}^d\}$ (see figure 4). For each $t \in \mathcal{T}^d$, we define the descending fluctuations variables $D_{t,k}^f$ by

$$D_{t,k}^f = \frac{D_t^k - \mu_t^d}{\sigma_t^d}$$

where $k \in \{1 \ldots, 23\}$ and $t \in \{0 \ldots, \mathcal{T}^a\}$. The descending fluctuations variables $D_{t,k}^f$ measures the deviations of the sunspot descending phases D_t^k to the descending mean μ_t in standard deviation σ_t^d units. Surprisingly, the histogram of the aggregated descending observed fluctuations, also, shows a data collapse to the universal nonparametric BHP pdf (see figure 6). In particular, the histogram of the descending fluctuations variables $D_{t,k}^f$ do not follow a gaussian distribution, exhibiting heavy tails and a universal non-zero skewness. The highest observed positive fluctuation $D_{t,k}^f$ is equal to 3.262 and the lowest observed negative fluctuation $D_{t,k}^f$ is -2.153 showing the asymmetries of the histogram. We get an estimator for the sunspot numbers

$$D_t^k = \sigma_t^d D_{t,k}^f + \mu_t^d \tag{3}$$

using the descending mean μ_t^d and the descending standard deviation σ_t^d, as obtained in figure 4, and noting that $D_{t,k}^f$ follows the universal nonparametric BHP pdf. In figure 4, we observe that the highest ascending means μ_t^d occur together with the highest standard deviations σ_t^d, for values of t close to 0. Hence, by equation (3), the highest sunspot numbers D_t^k occur for values of t close to to zero.

4. Conclusions

We analyzed the famous Wolf's sunspot numbers. Following Bramwell, Holdsworth and Pinton (4), we defined and analyzed the sunspot numbers fluctuations. We discovered that the distribution of the sunspot number fluctuations, for both ascending and descending phase, is close to the universal Bramwell-Holdsworth-Pinton (BHP) distribution. Our result reveal an universal feature of the Wolf's sunspot numbers. In particular, the histograms of, both, the ascending and descending fluctuations variables do not follow a gaussian distribution, exhibiting heavy tails and a universal non-zero skewness. For a given time of the ascending or descending phase, our result, also, gives an estimator for the probability of any given measurable set of sunspot numbers.

We would like to thank Peter Holdsworth and Henrik Jensen for showing us the relevance of the Bramwell-Holdsworth-Pinton distribution. We would like to thank an anonymous referee for pointing us the different character of the ascending and descending phases of the sunspot numbers cycles and, also, the importance of the different cycle length durations. We would like to thank Robert Wilson for giving us very useful information on the sunspot numbers.
We thank the Programs POCTI and POCI by FCT and Ministério da Ciência, Tecnologia e do Ensino Superior, and Centros de Matemática das Universidades do Minho e do Porto for their financial support."

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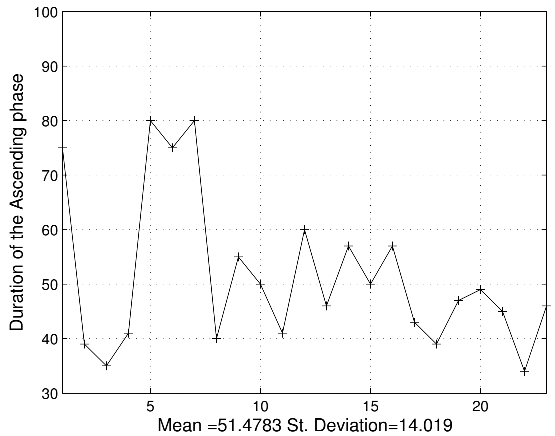

Fig. 1.— Duration of the ascending phases a_k of the sunspot cycles.

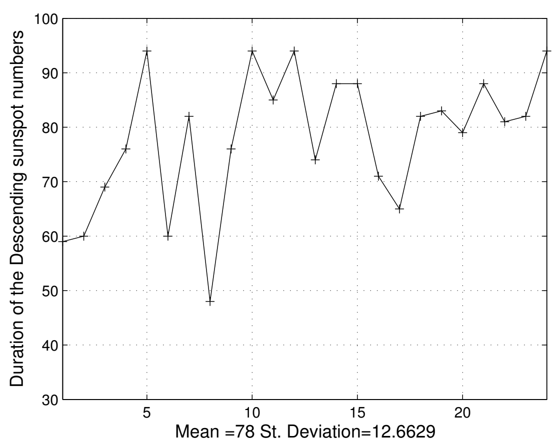

Fig. 2.— Duration of the descending phases d_k of the sunspot cycles.

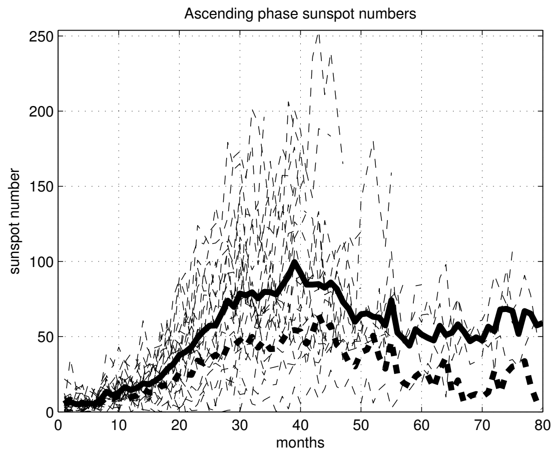

Fig. 3.— Ascending phases A_t^k of the sunspot cycles and respective mean (full line) and standard deviation (dotted line).

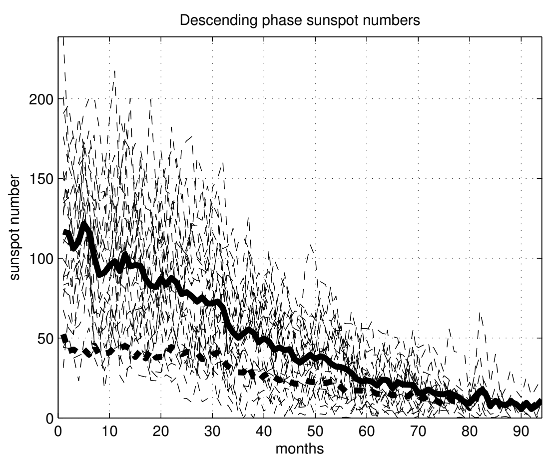

Fig. 4.— Descending phases D_t^k of the sunspot cycles and respective mean (full line) and standard deviation (dotted line).

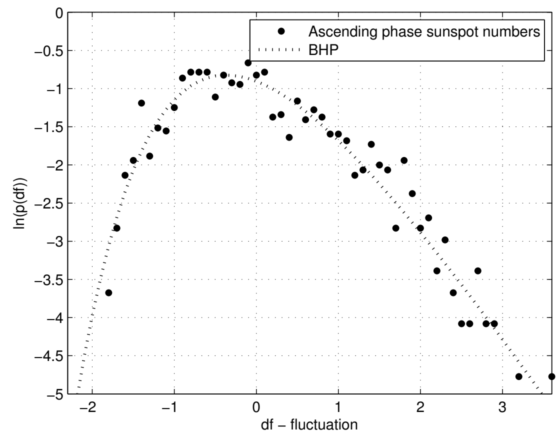

Fig. 5.— Histogram of the aggregated ascending fluctuations $A^f_{t,k}$ of the sunspot cycles.

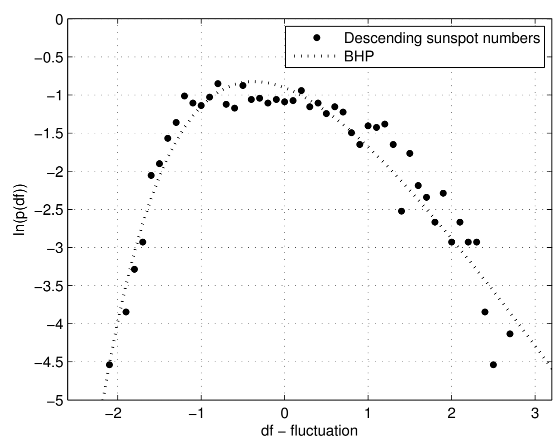

Fig. 6.— Histogram of the aggregated descending fluctuations $D^f_{t,k}$ of the sunspot cycles.