Physics and Computation:

Essay on the unity of science through computation

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Abstract

S. Barry Cooper and Piergiorgio Odifreddi have written the most interesting articles in our times on the philosophy of computing. In this paper I will try to reconcile their own views with established science and scientific criticism. I will take [3] as the main reference containing pointers to many ideas exposed in previous work of the same authors.

Computability Theory has been considered a corpse for mathematicians who did forget the *old debate about whether computability theory has useful consequences for mathematics other than those whose statements depend on recursion theoretic terminology.* In this context, HYPERCOMPUTATION is *a forbidden word* because *it is not implementable*, as foundational criticism says, although mathematicians do not mind to explore Turing degrees such as

$$K^{K^{K^{\cdots}}}$$

We explore the origins of this criticism and misinterpretations of concepts such as super-Turing computational power.

To make the discussion opened for all generations of mathematicians and physicists we developed our argumentation in a basis of a language of the late sixties and early seventies, decade of decline of the most enthusiastic Debates in Science: in Mathematics, in Physics, in Cosmology, etc., the times of Radio Programs by Sir Fred Hoyle, the times of the phone calls at the middle of the night between Sir Roger Penrose and Stephen Hawking, the times of the solution of Hilbert's Tenth Problem by Martin Davis, Hilary Putnam, Julia Robinson, and Yury Matiyasevich, etc.

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Contents

1 Universe and universes, Reality and realities, where it is shown that there is not such a difference between the real world and models — Stonehenge as calculator and Stonehenge as calculator with oracles.

The Astronomer Sir Fred Hoyle proved in [16,17] that Stonehenge can be used to predict the solar and the lunar eclipse cycles. It *doesn't really matter* whether the Ancients — the Celts — used or not this huge Monument (mainly the structure of what is called Stonehenge I) to predict the eclipse cycles, but it matters to me that we, in our times, can use Stonehenge I to make good predictions of celestial events like the azimuth of the rising Sun and of the rising Moon, or that we can use this Astronomical Observatory as an eclipse's predictor (see also [24] for a short introduction). One important structure to this task is the alignment of the Heelstone with the summer solstice and the circle of Aubrey holes, made of 56 stones, buried until the XVII century, and discovered by John Aubrey.

We make use of three counters (a 3-counter machine with bounded resources) for the task: the first counter, one little stone, counts the days of the year along the circle of Aubrey holes; the second counter counts the days of the lunar month; finally a third counter takes care of the Metonic cycle, in which the same phases of the moon are repeated on the same date of the year to within an hour or so after a period of nineteen years, a fact discovered by Meton around 430 B.C. but it is believed to have been known earlier — in other words it counts along the cycle of the lunar node, one of the intersection points of the ecliptic with the Moon's orbit.

Since $56 \times \frac{13}{2} = 365$, the first counter has to move two places — two Aubrey holes — each 13 days (one place per week roughly speaking), counterclockwise; in a similar way, since $56 \div 2 = 28$, the second counter is allowed to move two places per day, counterclockwise. When the two counters meet at the same hole an eclipse becomes possible, but only if the Sun and the Moon meets close to the lunar's node — intersection point of the ecliptic and the Moon's orbit. This point is represented by the third counter. Thus the three counters have to meet at the same hole (more or less). This third little stone counts along the Metonic cycle: $56 \div 3 = 18.67$ (very close to the true value 18.61, this is the most strange coincidence), meaning that it has to move 3 places — 3 Aubrey holes — per year, clockwise.

Thus the game — and the Rite — of the three stones around the circle of Aubrey holes — like the tokens in a Petri net — allows to predict the solar and the lunar eclipse cycles (seminal paper was published in Nature by the Archeologist Gerald Hawkins in [13], but mathematical calculations were done by Hoyle, years after). What do we have? A 3-counter machine with finite memory, which is equivalent — abstracting from bounded resources — to a Turing machine. Or else, think this way: a 3-counter machine implements an eclipse cycle predictor using arithmetic modulo 56. Is it not simple? — A quite straightforward algorithm is implemented by a special purpose machine, directly implemented on a general purpose machine. Now, the question: when playing with counter machines do we abstract the algorithm to see the Sun, a physical body, and the Moon, another physical body, and (the words are taken from Hoyle) a *holy spirit*, the lunar node, playing a dance — a macabre dance (if this paper was not to be a short account, then I would have added a Celtic invocation of the gods) — in the sky, projected into the celestial sphere? Yes, of course, if we have a Rite associated to the *ballet*.

So, *the Druids were "aware" of the Turing model of computation...* But, the counters, with time... loose accuracy (like making calculations with the real numbers [1]). This is not the whole story! Once in a year, the Sun rises over the Heelstone. Some auxiliary stones (*the post holes*), to one side of the Heelstone, can be used to fine tune the counters: the site of the rising mid-summer Sun move to the north and then back to the south, allowing to fine tune the Sun's counter by observing from the center through the post holes its maximum azimuth. Some auxiliary stones also help to fine tune the second counter. All

[1] That is also what Barry Cooper and Piergiorgio Odifreddi consider in *What the Turing Model Delivers* when they write *That there are sufficient indicators, of both a practical and theoretical nature, for us to look for a model for the Universe based on presentations in terms of real numbers.*

the stones lie there is Salisbury, in the big circle. Observations operate like oracles of the Sun and of the Moon. Thus the Master, in the center, can substitute the accurate algorithm of the eclipse by a less accurate algorithm, together with an oracle for the Sun and a second oracle for the Moon. Thus, *the Druids were "aware" of the Turing model with oracles...*

The curious thing is that the putative referents to the real objects make a simple model of solar system dynamics. We have a Reality in the sky, and another reality in the big circle. In this sense we are very close to *A New kind of Science* by Wolfram in [43]. The algorithm is captured by the real world in the sense that the real world embeds the algorithm.

THESIS 1 — Computers exist in Nature when we abstract the physical entities. (This fact will be referred to as *Galileo's principle of natural computation.*)

Barry Cooper and Piergiorgio Odifreddi in [3] explain this fact digressing on *A Closer Look at the Turing Model*: *we expect that the Turing model supports the (in)computability in Nature in the sense that nature embeds the Turing model in a way or another*. But for our authors (and we will discuss this aspect later), (in)computability sounds more like an intrinsic limitation of knowledge about the Universe than a hypercomputation manifesto.

The word *Universe* that these authors often use has a major disadvantage. When used alone, without specification of the model we have in mind, it conveys the impression that we know the true nature of the Universe. A *universe* is simply a model of the Universe. The word *universe* has the further advantage that it may be used freely and loosely without any need to remind ourselves constantly that the Universe is still mysterious and unknown. The same argumentation applies to *Reality* and *realities.*

Our model... our Stonehenge model of (a fragment of) the Universe is not so bad, and it really tells about the Universe in the sense it can be found in the intersection of *all models* of the Universe. That is why, I think, that Computation Theory is not so far from Physics and, probably, with the advent of Quantum Computation, a serious transference of competence from Mathematics to Physics will occur.

Stonehenge as model of the Universe can be also used as a calculator and computer, just by doing *bounded arithmetics* and, why not, to implement some more sophisticated algorithms. Moreover, Stonehenge *implements natural phenomena*, the complex movements of the Sun and the Moon in the sphere. Stonehenge possess' the means of consulting oracles in *Nature* itself. Oracles here handle some incomputabilities in nature, of the same kind described above, that do not come out from the model, but come out of observations.

The way we abstract the Turing model, being it through Recursion Theory or a different formalism is not really important. Computability Theory becomes a *corpse* to the mathematicians who will be continuously trying to develop more and more mathematical tools to dig, sometimes as our authors say, *too* deep into mathematics. I have a first disagreement: Computation Theory is a recent field of Mathematics — it will never be as *deep* as Analysis with their 400 years of history, at least in short term.

This does not mean that modern mathematics does not play a puzzling role in the development of knowledge. If Shimura-Taniyama conjecture to analytic number theorists was a stupendous statement wrt *Fermat's Theorem provers* — it looked like saying (see [4]) *if in a room there exist* 7 *women and* 7 *men, and if we are aware of* 7 *marriages, then a bijection between men and women would be the case* –, then proofs of termination would be more theatrical, even to analysts: take number 9 and write it in base 2, as $1 \times 2^{2+1} + 1 \times 2^0$; Achilles arrives and replaces all 2 by 3 writing $1 \times 3^{3+1} + 1 \times 3^0$; now, the turtle arrives and subtracts 1, making the amount $1 \times 3^{3+1}$; Achilles, very secure of his job, rewrites everything in base 3, and substitutes 4 for 3 making $1 \times 4^{4+1}$; the turtle subtracts 1; then Achilles rewrites the result in base 4, making $3 \times 4^4 + 3 \times 4^3 + 3 \times 4^2 + 3 \times 4 + 3$ and replaces the 4 with 5. Does this process terminate? Ordinal theory — can you imagine? — can be used to prove that this process indeed terminates in finite time and the turtle wins – do you believe? The termination is due to the fact that ordinals are well-ordered. We simply substitute the ordinal ω for the numbers 2, 3, 4, ... that were used as a base: $\omega^{\omega+1} + 1$, $\omega^{\omega+1}$, $3\omega^{\omega} + 3\omega^3 + 3\omega^2 + 3\omega + 3$, ... It is easy to see that we have a strictly decreasing sequence of ordinals and that such a sequence is necessarily finite, which means that the tortoise wins (see [10]). This is the kind of deep thoughts that Modern Mathematics brings about.

But, removing from our consideration the interesting meeting points of Modern Mathematics, in order to get deep into mathematical reasoning we still have to move to classical mathematics. The authors, recognize this fact somewhat implicitly when they write:

> *... to outsiders classical computability had become hazardous and, even by the standards of fundamental scientific research, lacking predictability of outcome; in which mathematical applications depended on recursion theoretic terminology; and in which the undoubted contribution to theoretical computer science and constructive mathematics did not depend on the sort of things that recursion theorists currently occupied themselves with.*

We end this section with a question: do these incomputabilities come out of an unpredictable behaviour of the model or do they come out of a really essential incomputability in nature — a hypercomputational character of some physical phenomenon, as Sir Roger Penrose was looking for in [28–30]?

Was Cristopher Moore the first to observe that essential chaos exist in nature. We were accustomed to think of chaos as departure from initial conditions. Moore proved that essential chaos exists, a chaos that is so essential (like Black Holes in Hawking-Penrose's theory) that the infinite precision in the initial conditions will not remove it. The proof is straightforward: he shows that the collection of dynamical maps contains many instances of simulations of Universal Turing machines. In other words, any property that a map can have, like being injective, or onto, or having an infinite domain, or having an infinite range, or being total, is undecidable unless it is trivial (corresponding to the empty set and to the set of all recursive maps). Now, in terms of dynamical systems, these questions concern basins of attraction. These basins are in general non-recursive, i.e., there is no hyper-algorithm that will tell us whether or not a point is in them. In fact, we can even recall Theorem 10 from [23]:

Proposition 1.1 (Moore's undecidability theorem) *The following questions about discrete-time [continuous-time] dynamic systems are undecidable.*

(1) Given a point x and an open set A, will x fall into A?
(2) Given a point x and a periodic point p, will x converge to p? Will a dense set of points converge to p?
(3) Is the set of periodic points on a given cylinder infinite? Dense?

Thus returning to our authors: to what kind of incomputability should we search for in Nature?

POSSIBLE INTERPRETATIONS:

- **(A) Should we look for *partial information* as incomputability (like hidden variables in the Paris School of Quantum Mechanics),**
- **(B) Should we look for *essential chaos*, removable by means of natural "hyper-machines" — a better saying is to control *unpredictability* in Nature –, or**
- **(C) Should we look for *hypercomputational phenomena* in the Universe (in the sense of [28–30]).**

Article [3], written by top leading scientists in Computer Science raises many questions about the nature of Computability and its Philosophy. This is, I guess, a very important prospect to deliver Computer Science to other Sciences.

2 The n-clocks machine and computability in Nature, where it is shown that making calculations in the real world is not a new task, not even a difficult one.

Let us return to the Stonehenge's counters. The n-counter machine for any n greater than 1 has a very well known property.

Proposition 2.1 (Universality of n-Counter machine) *There is a* Turing universal 2-*counter machine.*

Now let me speak about the nice feature of the n-counter machine. Everybody knows that counter machines are *horrendous*. Students play with the Turing machine to get acquaintance of a model of computation: exercises like to describe Turing machines for calculating the (a) sum, (b) product, (c) division, ... are a common place in classes of Computability Theory. But, we know how troublesome these programs for counter machines are! Otherwise, a computer program, developed in a high-level computer language like C^{++}, to predict the eclipse cycles is cumbersome too! But, look at the miracle, the Stonehenge computer, a 3-counter machine, computes this cycle very trivially. We have to wait a long time... But the slowdown is LINEAR. It is irrelevant.

Thus, in the Stonehenge case, the counter machine is a kind of natural computor.

I will consider now a more sophisticated machine that is not well known — I presume that it is really very obscure — that I fetch from a paper by Joe Killian and Hava Siegelmann (see [18]). This model will reveal its extreme beauty when regarding *natural computing* [2] in Stonehenge. The model of computation is the *alarm clock machine*. By an alarm in the physical world we can consider, e.g., an astronomical ephemeris, like a conjunction of planets, or the reaching of a perihelion, or an eclipse, or many other things in macroscopic or microscopic worlds. Could well be a signal sent to Jupiter by the Monolith after its discovery by men on the Moon in the well known movie "2001, A Space Odyssey". This kind of automaton was first introduced in [18] for an *ad hoc* purpose, and it is called n-alarm clock machine (or just n-clock machine) consisting on a transition function $\delta : \{0,1\}^{5n} \to 2^{\{delay(i), lengthen(i):1\leq i\leq n\}\cup\{halt\}}$. The fact that δ's domain is $\{0,1\}^{5n}$ means that δ's input is simply the information of which alarm clocks have alarmed in the last 5 time steps [3] and when they did so. δ's output is simply which clocks to delay, which clocks to

[2] I know that natural computing is nowadays interpreted as DNA computing, membrane computing, etc. ...

[3] The number 5 is considered here just because we know the existence of a universal n-clock machine with constant 5, Of course, it is not proved that a universal n-clock machine exhibiting a lesser structural constant does not exist.

lengthen, and whether the machine halts or not.

The input to A consists of $((p_1, t_1), \ldots, (p_n, t_n))$, where p_i denotes the period of clock i, and time t_i denotes the next time it is set to alarm. For notation ease, we keep arrays $a_i(t)$, for $t \in \mathbb{N}$ and $1 \leq i \leq n$, with every entry initially set to 0. At time step T, for $1 \leq i \leq n$, if $t_i = T$, then $a_i(T)$ is set to 1 and t_i is set to $t_i + p_i$. This event corresponds to clock i alarming. δ then looks at $a_i(t)$, for $1 \leq i \leq n$ and $T - 5 < t \leq T$, and executes 0 or more actions. Action $delay(i)$ sets t_i to $t_i + 1$, action $lengthen(i)$ sets p_i to $p_i + 1$ and t_i to $t_i + 1$, and action *halt* halts the alarm clock machine. We make one final restriction to the behavior of an alarm clock machine: when its transition function is applied to a vector of $5n$ 0's, then it outputs the empty set of actions. Intuitively, this corresponds to demanding that the machine must be asleep until it is woken.

The role of the alarm clocks of the alarm clock machine is to store information on the frequency at which they alarm. In the same way as in Turing machines the tapes are potentially infinite, but at any given instant only a finite amount of information is actually stored on the tape is finite, the period of the clocks may increase unlimitedly, but any given instants all alarm clocks have a period limited by some given constant.

A n-alarm clock machine A is then a total function from $\{0,1\}^{5n}$ to a subset of $\{delay(i), lengthen(i) : 1 \leq i \leq n\} \cup \{halt\}$ that verifies $A(00\ldots00) = \emptyset$. Given a n-alarm clock machine M, an instantaneous description of M, also called a configuration, is a n-tuple $((p_1, t_1), (p_2, t_2), \ldots, (p_n, t_n))$, where $p_i \in \mathbb{N}$ denotes the period of the ith clock and $t_i \in \mathbb{N}$ denotes the next time at which the ith clock will alarm. A n-alarm clock machine does not receive as an input a sequence over a given alphabet. Instead, the input to the alarm clock machine consists of a n-tuple $((p_1, t_1), (p_2, t_2), \ldots, (p_n, t_n))$, where $p_i \in \mathbb{N}$ denotes the period of the ith clock and $t_i \in \mathbb{N}$ denotes the next time at which the ith clock will alarm, i.e., the input to the alarm clock machine is its initial configuration. To make precise we define the following items:

Given a n-alarm clock machine M, and an initial configuration of M, $((p_1, t_1), (p_2, t_2), \ldots, (p_n, t_n))$, the computation of M on the given configuration is a sequence of configurations that verifies

$$p_i(t+1) = \begin{cases} p_i(t) + 1 \text{ if } lengthen(i) \in \delta(t) \\ p_i(t) \qquad \text{otherwise} \end{cases}$$

$$t_i(t+1) = \begin{cases} t_i(t) + 1 & \text{if } delay(i) \in \delta(t) \text{ or } lengthen(i) \in \delta(t) \\ t_i(t) + p_i(t) & \text{if } t_i = t \\ t_i(t) & \text{otherwise} \end{cases}$$

Proposition 2.2 (Simulation of the n-counter machine) *For a n-counter machine that computes in time T, there exists a k-alarm clock machine that simulates it in time $O(T^3)$ with $k \in O(n^2)$.*

Proposition 2.3 (Universality of n-clock machine) *There is a* universal *n-clock machine, for some n.*

The reader will recognized that this kind of machine implements astronomical queries in the dynamics of the Solar System in a natural way, like Stonehenge token game implements the eclipse cycle in a natural way. We can add oracles at precise conjunctions of heavenly bodies. This machine is like the Turing machine, universal, with unbounded memory and suitable for thinking with physical bodies, e.g., in the Newtonian gravitational field. One idea to implement the heavenly machinery in Stonehenge in the big circle: take the orbit of each planet and expand it — the radius vector — in Fourier series, in the complex plane of its orbit, and consider the first n periods implemented by n tokens around the circle of Aubrey holes.

THESIS 2 — Computers exist in Nature by the simple observation of the sky. Moreover, the standard model — the Turing machine — can be described in such a way that it resembles the old astronomical observations in the sky by the Ancients. (This fact will be referred to as *physical principle of computation*.)

I got this idea, although in a quite different context, not related to Stnohenge or even to the clocks machine, from Hava Siegelmann and Eduardo Sontag, and their hypercomputational dream settle in Science (the Journal) by Hava Siegelmann (see [38], end of Book, for a more detailed account). As our authors say, the experience I described above of celestial conjunctions is like looking to the concept of incomputability as *action at a distance* in the time of Newton's vision of the Universe. Oracles are needed to fine tune the system once in a while not only to remove errors (real to natural numbers), but also to *remove Moore's unpredictability.* This is exactly how Barry Cooper and Piergiorgio Odifreddi start their curious paper, discussing our deterministic universes, our Laplace demons. In theirs *An Historical Parallel*, in 2 to 3 pages, they question about items (A) to (C) above, but in a very abstract level, in my spirit not distinguishing very well the three items of the incomputability riddle.

I think that, here, our colleague Warren Smith gave a very good contribution in [40], when (this is ridiculous!) he says that although we are able to show that Newton's gravitation admits a non-computable orbit (a kind of incomputability of the third kind (C)), special relativity removes it from consideration. Dear reader, written in a different way the paper by Warren tells us that *the discovery of a non-computable orbit in Newtonian mechanics refutes Newtonian gravitation theory according with the Church-Turing thesis* in the same way, philosophically speaking, as the curvature of light rays from distant stars in the proximity of the Sun refutes it. Warren's paper tells us that the Church-Turing thesis refutes classical Newtonian gravitation. As if Einstein have had a computational reason to create a new theory of gravitation. Or else we have to disclaim the Church-Turing thesis. I do not know if Warren is aware of this philosophic riddle.

If the reader take a closer look at Piergiorgio Odifreddi's text books ([26,27]) on the Church's Thesis, from page 101 to page 123 in [26], he or she will find different formulations of the classical Church's Thesis, such like Kreisel's Thesis M (for *mechanical*) or like Kreisel's Thesis P (for *probabilistic*), and so on. Here, at this precise point of his text without rival (!), he wrote *In the extreme case, any physical process is an analog calculation of its own behaviour.* And... Piergiorgio Odifreddi adds a quite interesting footnote:

> *In this case, Church's Thesis amounts to saying that the universe is, or at least can be simulated by, a computer. This is reminiscent of similar tentatives to assimilate nature to the most sophisticated available machine, like the mechanical clock in the 17th Century, and the heat engine in the 19th Century, and it might soon appear as simplistic.*

In fact, Thesis P states that *any possible behaviour of a discrete physical system (according to present day physical theory) is recursive.* Thus Warren Smith disproofs this statement. There exists a Newtonian non-computable orbit. It is not relevant that Relativity Theory removes this pathology: no one would ever believe a few years ago that Thesis P would not be valid... Or is it still valid? Well, physicists say, we don't have point masses or two masses can not come as closer as we want. I argue that these are qualitatively physical aspects that are *not in* the formulation of Newtonian gravitation. I will restate saying that no one would ever believe a few years ago that Thesis P would not be valid even for the abstract gravitation theory taught in physical courses. [4] Please, let me know about a Student's Manual that considers from scratch the problem of two bodies with non-zero volume. If then Professor X shows that the n-spherical-body problem gives rise to a non-computable orbit, physicists will say that planets are not really spheres. Is it not the same problem as with Black Holes, and the story of the meeting point between the collapsing star and Black Hole formation? There is in here something to be better explained.

Thus when authors write:

> *Fortunately, there is another approach — let's call it the "mathematical" approach — which renews the link to Newton. This is a direction rooted*

[4] The most comprehensive study I know is a Treatise about stability of a spacecraft, considering 2-body dynamics, on one side the spacecraft with non-zero dimensions and on the other side the Earth just substituted by a its gravity center.

in the old debate about whether computability theory has any useful consequences for mathematics other than those whose statements depend on recursion theoretic terminology,

I would have added: *the new debate about whether computability theory has any useful consequences for physics.*

Then we can say that Nature has an algorithmic contents: it is greater than the algorithmic contents of the Solar System, greater than the algorithmic contents of the system Moon-Sun-Earth, graeter than the algorithm contents of Stonehenge I. If Stonehenge IV would have been built than, certainly, it would implement the n-clock machine. Does the Universe, or just the universes, have an algorithm contents greater than the algorithm contents of Stonehenge IV, abstracting from bounded resources?

3 Predictability, platonism, and the discovery of Neptune, where it is shown that limits to hypercomputation exist no matter it isn't real.

I would like to start this section saying that the removal of *action at a distance* from physics in contemporary science is like the removal of the Rite in Stonehenge I: for Newton action at a distance was done by means of (a) God (too): space is the *Sensorium Dei* by means of which He stabilizes the system. Newton himself writes in his *Opticks*:

> *... can be the effect of nothing else than the Wisdom and Skill of a powerful ever-living Agent, who being in all Places, is more able by his Will to move the Bodies within his boundless uniform* Sensorium*, and thereby to form and reform the Parts of the Universe, than we are by our Will to move the Parts of our own Bodies. And yet we are not to consider the World as a Body of God, or the several Parts thereof, as the Parts of God. He is an uniform Being, void of Organs, Members or Parts, and they are his Creatures subordinate to him, and subservient to his Will; and he is no more the Soul of them, than the Soul of Man is the Soul of the Species of Things carried through the Organs of Sense into the place of its Sensation, where it perceives them by means of its immediate Presence, without the Intervention of any third thing.*

Removal of the Rite in Stonehenge I is like the Laplacian removal of the *Sensorium Dei* from Newton's space.[5] But with a *Sensorium Dei* or without

[5] As the reader can see, Newtonian space like Descartes' substancial space was not empty but the *Nervous System of God.*

it, Warren Smith proved the existence of non-computable orbits: an incomputability of the third kind (C), although the proof is based on some abstract physical systems related to the Poincaré Conjecture, proved by Jeff Xia in [42], and others for particular cases.

Barry Cooper and Piergiorgio Odifreddi raise the question:

> *Why should those without a direct career interest care whether actual incomputability (suitably formalized) occurs in Nature? Even if it did occur, for all practical purposes, how would it be distinguishable from theoretically computable but very "complex" phenomena? Whether chaotic phenomena — such as turbulence — involve complexity or incomputability is interesting, but does it really "matter"?*

This questions are also related with Barry Cooper's ideas stated in his later nineties paper ([1]). We think that the answer to this question is not easy. On contrary, this is singular and unexpected: differential equations do exist, with computable coefficients, given computable initial conditions, which can not be numerically solved via deterministic or non-deterministic methods by a digital computer: their solution are beyond the Turing limit. Lady Marian Pour-El and Jonathan Richards provided examples in [33–35]. Physicists say, however, that these examples have initial conditions, or boundary conditions, which are not smooth enough to describe real physical situations. Are then (we ask) all *physical laws* digitally reproducible by a digital computer? If true, then we may talk about non-computable functions as those functions that can not be known through numerical analysis by means of digital computers, although they satisfy very simple differential equations — and the knowledge about these functions is fundamental in Mathematics. Calculating positions of planets (ignoring some possible incomputabilities suggested in [40]) was, in fact, a problem of precision: *we don't have real numbers in it.* Intrinsic non-computable functions of Pour-El and Richards are of a different kind. In [34] (dealing with computability in models of physical phenomena), Marian Pour-El and Richards wrote:

> *We consider first the three-dimensional wave-equation. It is well known that the solution $u(x, y, z, t)$ is uniquely determined by the initial conditions u and du/dt at time $t = 0$. We ask whether computable initial data can give rise to non-computable solutions. The answer is yes. [We give] an example in which the solution $u(x, y, z, t)$ takes a non-computable value at a computable point in space-time.*

Do we have a model to classify such sources of uncomputability found in [33–35]? No, we don't. Do you imagine an equation — Poisson's equation — as simple as

$$\psi(x, 0) \;=\; f(x),$$

$$\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial t^2} \;=\; 0,$$

having a non-computable unique solution (non-computable in the sense of conventional computable analysis): there exists not a program such that giving the values of computable numbers x and t with increasing precision will provide $\psi(x,t)$ with increasing precision, despite existing such a program for the function f.

Sir Roger refutes these examples as useful to a forthcoming *Non-computable Physics*, since boundary conditions or initial conditions involved are not smooth enough. Penrose in [28] stresses this fact before considering the (non-computable) ultimate physical theory to come and the human mind:

> *Now, where do we stand with regard to computability in classical theory? It is reasonable to guess that, with general relativity, the situation is not significantly different from that of special relativity — over and above the differences in causality and determinism that I have just been presenting. Where the future behaviour of the physical system is determined from initial data, then this future behaviour would seem (by similar reasoning to that I presented in the case of Newtonian theory) also to be computably determined by that data (apart from unhelpful type of non-computability encountered by Pour-El and Richards for the wave equation, as considered above — and which does not occur for smoothly varying data). Indeed, it is hard to see that in any of the physical theories that I have been discussing so far there can be any significant "non-computable" elements. It is certainly to be expected that "chaotic" behaviour can occur in many of these theories, where very slight changes in initial data can give rise to enormous differences in resulting behaviour. But, as I mentioned before, it is hard to see how this type of non-computability — i.e. "unpredictability" — could be of any "use" in a device which tries to "harness" possible non-computable elements in physical laws.*

Now, what is the consequence of this to Science? Even for *complex phenomena* like the dynamics of the atmosphere we have strong methods of numerical integration. We take Navier-Stokes equation and we consider (a) spherical coordinates, (b) that the Earth is not an inertial reference frame, (c) boundary conditions around east North-America's shore and West-Europe's and North-Africa's coast, and we (a) presume that such differential equations are integrable by numerical methods and (b) that a prediction of the Weather for *tomorrow* can be obtained *before tomorrow*. Thus we still have (a) computability considerations and (b) computational complexity considerations. Philosophically speaking, we are turned to models of Nature which are of a predictable intrinsic nature. Science in this way is used (a) to make a synthesis of our knowledge about the Universe and (b) to forecast future events. I think that the answers to Barry Cooper and Oiergiorgio Odifreddi are "yes", "we don't know", and "yes". A non-computable world like the model intended by Sir

Roger Penrose would have a quite different meaning. Assuming that no more computational power is added to computers, we wouldn't have general predictions. The model would be looked like a God: suddenly a pattern formation occurs out of that model and some sophisticated computer programs would be able to trace and forecast its trajectory, like a hurricane that although can not be exactly predicted (but expected) can be followed, either by satellites, either by computer programs. A Non-computable Science would be more like *a painting* in the National Gallery — to look at with respect, admiration, and fascination, but although interpreted by many, not really being interpreted by none besides digressions and elucubrations of the critics (probably it will fulfill Susan Sontag in hers *Against Interpretation*). May well be that Barry Cooper and Piergiorgio Odifreddi questions are more likely to be as follows: (a) are our Contemporary Science a pattern of a non-computable model? or (b) do we already have a Non-computable Science, hidden in our theoretical achievements? or either (c) Non-computable Science is no more than contemporary fiction, motor and product of the creative process, like the *stone* was for alchemy. No matter the true answer, they make the concluding statement that:

> *Our model says nothing about the mystery of material existence. But it does offer a framework in which a breakdown in reductionism is a commonplace, certainly not inconsistent with the picture given of levels we do have some hope of understanding. It can tell us, in a characteristically schematic way, how "things" come to exist.*[6]

We also know that *modern science* loosed some identity. There is just ONE Newtonian gravitation theory,[7] but with the advent of General Theory of Relativity, physicists realized that Einstein's beautiful field equation

$$R_{ij} - \frac{1}{2}\, g_{ij}\, R \;=\; \kappa\, T_{ij}$$

could be replaced by different field equations delivering the same realities, delivering the same predicted observations of our Universe. Most probable an non-computable model will deliver also a class of similar observations of the Universe. E.g., is Hoyle's or Hoyle-Narlikar's field of creation *ex nihilo* non-computable? This is not philosophy, since Hoyle's field of creation out of nothing is hard mathematics, being it refutable nowadays, not accepted by the

[6] The reader can have a look at the sixties Niels Bohr's Cosmic Equation, that was a source of explanation of how the Universe came into existence.

[7] In fact, for some time, physicists were tempted to define the law

$$\frac{1}{r^{2.0\cdots025}}.$$

Ridiculous, isn't it? But it worked for a few years, when physicists loosed their faith for reasons that will become clear soon.

scientific community as the Big Bang Theory is — the standard model. It holds the same observations as Einstein field equations up to some cosmological and astrophysical observations.

Hoyle arrived at the alternative equation

$$R_{ij} - \frac{1}{2}\, g_{ij}\ R\ + C_{ij} =\ \kappa\ T_{ij}.$$

Associated with the creation tensor C_{ij} was a vector field parallel to a geodesic at each point of the homogeneous and isotropically expanding universe. The field was written

$$C_m = \frac{3c}{a}\ (1, 0, 0, 0),$$

where a is a constant. Hoyle then showed that the solution of the field equations would be given by a metric with space of zero curvature.

Bondy, Gold, and Hoyle used the word creation rather than formation, just to express the existence of matter where none had been before.

With the Hoyle-Bondi-Gold's model we can evaluate the amount of matter being created at any step of time. But can we predict the point in space where a proton (Hoyle guessed that the spontaneous creation of matter might possible be generated in the form of neutrons) will next appear? This is an example of how a non-computable aspect of a theory (we can not even guess a distribution of matter created [8]) can deliver also computable trajectories of our Universe.

[8] Edward Harrison explains these features in [12]:

> *There are two kinds of creation: creation of the universe and creation in the universe. On one hand, we have creation (as in cosmogenesis) of the whole universe complete with space and time; on the other, we have creation of things in the space and time of an already existing universe. In the Big Bang universe, everything including space and time is created; in the steady-state universe [of Bondi, Gold, and Hoyle], matter is created in the space and time of a universe already created. Failure to distinguish between the two violates the containment principle... The steady-state theory employs creation in the magical sense that at certain place in space at a certain instant in time there is nothing, and at the same place a moment later is something. But the creation of the universe has not this meaning, unless we revert to the old belief that time and space are metaphysical and extend beyond the physical universe; in that case, creation of a universe is in principle the same as the creation of a hazel nut. But in fact uncontained creation (cosmogenesis) is tottally unlike contained creation. Cosmogenesis involves the creation of space and time, and this is what makes it so difficult to understand.*

Barry Cooper and Piergiorgio Odifreddi recognizes these different presentations of the Universe stating that:

> *we look for a mathematical structure within which we may informatively interpret the current state of the scientific enterprise. This presentation may be done in different ways, one must assume, but if differing modes of presentation yield results which build a cohesive description of the Universe, then we have an appropriate modeling strategy.* Going more deep into the quantum they say ... *non-locality was first suggested by the well-known Einstein-Podolsky-Rosen thought experiment, and again, has been confirmed by observation. The way in which definability asserts itself in the Turing universe is not known to be computable, which would explain the difficulties in predicting exactly how such a collapse might materialize in practice, and the apparent randomness involved.*

The n-clock machine can be implemented with bounded resources in Stonehenge using colored stones, a color for each clock, 5 colored tokens for each clock. [9] It would have made Stonehenge a huge Observatory (although many existent stones — like the post holes — can handle a large number of calculations that, despite the non-existence of a useful — to the Celts — implementation of the n-clock machine, make Stonehenge I and II a rather huge Astronomical Observatory). But I am going to talk now about a feature that can not be implemented in Stonehenge: the discovery process!

After Herschell discovery of *Uranus*, deviations from computed orbit, using Gauss methods, produced more and more observations of the new slow planet, enforcing calculations of more and more accurate orbits. But the new planet always failed to meet the computed orbit: the n-clock machine programmed with the beautiful Newtonian gravitation system refused to explain *Uranus'* orbit; our Obervatory at Stonehenge fails too to meet *Uranus'* potential conjunctions.

There are two attitudes. First one, to accept a decadence period, the Druids failed the prediction of planet cycles — Hoyle admits a period of mental decadence —, Stonehenge fails as Observatory, but the remembrance of the glorious Stonehenge compelled the Celts to built the Stonehenge's Sanctuary, named Stonehenge III, the colossal construction of central 3-liths; second one, to accept that the Newtonian law is wrong and search for the "true" law of gravitation.

But, as we know, it was to early to reject Newtonian theory of the Universe. Empirists failed. The law $\frac{1}{r^2}$ failed... But Leverrier and Adams, one in Paris the other in London, admitted that a new planet existed — later called Neptune

[9] It would be like a Calendar with many entries. It reminded me, at a first glance, Edward M. Reingold and Nachum Dershowitz' *Calendrical Calculations*, the Book.

— to justify the *true* (Newtonian) law of gravitation, to justify departure from predicted orbits. This can not be done by a computer program... Can it? Well, Herbert Simon (a more comprehensive study can be found in [36]) says that it can! — at least Kepler's laws can be rediscovered by computer programs (given Tycho Brahe's data) –, ... but not predicting a new planet. What is the difference (if the planet was not found) between predicting a planet and replacing Newton's law by another law, being it computable or not? Is the discovery of a new planet a kind of *incomputability removal*? This example is close to Penrose super-program conceived to mock a mocking bird, i.e., capable of reproducing any kind of scientific achievement.

What did the scientists about Leverrier and Adams predictions? Simple: they rejected them, they simply didn't believe. Is it not an amazing fact of strong computability of scientists mind: *if some hypothesis not in the system is suggested, then it should be immediately rejected*. E.g., Airy rejected Adams several times: how, I would like to go to Greenwich and knock at the door to hear him saying NO! As Morton Grosser tells the story in [11], *Airy was an extreme perfectionist, and he divided the people around him into two groups: those who had succeeded and were worthy of cultivation, and those who had not succeded and were beneath consideration [...] Adams solution of the problem of inverse perturbation was thus a direct contradiction of Airy's considered opinion. The Astronomer Royal's negative feelings were indicated by the unusually long time he waited before replying. Airy habitually answered his correspondence by return mail*. In Adams case he delayed the answer the much he could.

It would have been enough to look to the sky with the telescope, considering the calculations of Leverrier and Adams that Airy was aware of. It would have been enough if Sir Airy have agreed in doing so using calculated positions of Neptune.

Feel the pleasure of the following letter of Airy to Adams, that could have been a letter about any other relevant *thing* by any other illustrious scientist of our times:

> *I have often trought of the irregularity of* Uranus*, and since the receipt of your letter have looked more carefully to it. It is a puzzling subject, but I give it as my opinion, without hesitation, that it is not yet in such a state as to give the smallest hope of making out the nature of any external action on the planet [...] But [even] if it were certain that there were any extraneous action, I doubt much the possibility of determining the place of a planet which produced it. I am sure it could not be done till the nature of the irregularity was well determined from successive revolutions.*

In a further letter, Sir Airy writes to Adams:

> *I am very much obliged by the paper of results which you left here a few days*

since, showing the perturbations on the place of Uranus *produced by a planet with certain assumed elements. The latter numbers are all extremely satisfactory: I am not enough acquainted with Flamsteed's observations about 1690 to say whether they bear such an error, but I think it extremely probable.*

But I should be very glad to know whether this assumed perturbation will explain the error of the radius vector of Uranus. *This error is now very considerable.*

According with [11], on September 18, 1846, Leverrier wrote to Johann Gottfried Galle, assistant to Olaus Roemer. This letter reached Galle on September 23, and he immediately asked his superior, Johann Franz Encke, Director of the Berlim Observatory, for permission to search the planet. The same night Galle and d' Arrest found the planet: *that star is not on the map* — exclaimed d'Arrest; right ascension $22^h\ 53^m\ 25^s.84$ against the predicted value of Leverrier $22^h\ 46^m$. Although impressing this accuracy is smaller than Stonehenge's accuracy for the eclipse cycle.

I think that serendipity in the case of Archimedes' *Eureka!*, or in the case of Kepler's laws by Kepler, or Kepler's laws by Herbert Simon's program are different from the kind of discovery related to the existence of Neptune.

We are ready to introduce our

THESIS 3 — Hypercomputation as tool to supersede Natural laws and control them is beyond the limits of science (*principle of hypercomputation upper bounds.*)

I don't say that hypercomputation is to the discovery of Neptune, but that, citing Barry Cooper and Piergiorgio Odifreddi, *Science since the time of Newton, at least, has been largely based on the identification and mathematical description of algorithmic contents in the Universe. We will look at phenomena — primarily subatomic phenomena — which appear to defy such description.*

It seems that the hidden planet Neptune stands as a hidden variable to quantum mechanics defeating Copenhagen school and crediting Paris and De Broglie main hypothesis. This incomputability can be seen with the help of the n-clock machine.

We all know that pendulum clocks are quantum systems: each one has exactly two different energy levels, two oscillatory modes: one with the pendulum at rest and other with the pendulum oscillating in a stable orbit. We all know that clocks in the same wall propagate across the wall sound waves, together with their delays or advances, forcing the (coupled) clocks altogether, to a same delay or a same advance. In Stonehenge, this effect can not be seen between colored tokens, but on the human machinery that puts the little tokens in motion. Some people forget clocks [10] when they think about the quantum realm. Quantum mechanics in this way also applies to the macroscopic world (of course, not in the sense of making Planck's constant going to zero! [11]), in the sense of operators, eigenvectors and eigenvalues. It works like coordinated Druids working according the same teleological implicit thinking: a disagreement in the predicted Metonic cycle compels the Druids to add *a further token* to the game.

4 Algorithm contents, where it is shown that author's thinking differ in subtle items from other authors.

I disagree in a few statements stated in sections *Finitism in a Universe with Algorithmic Content* and *The Inseparability of Algorithmic Content, Complexity and Incomputability.* First Barry Cooper and Piergiorgio Odifreddi states that

> *... incomputability has about as much significance for a complete description of the Universe as it does for any other finite relational structure, such as a graph — that is none. In fact (see the discussion of Church's thesis in volume I of "Classical Recursion Theory") no discrete model — finite or otherwise — presents likely host for incomputable phenomena.*

We have at least two exceptions: Wolpert in [44] studies a discrete neural model with super-Turing capabilities, but with a transfinite number of neurons, and Pollack in [31] proved that a model of higher-order neural nets is at least universal. Other results on neural networks involve the real numbers. We have an idea that infinite automata can have super-Turing powers, even not involving the real numbers. Secondly, scientifically presenting the Universe with real numbers is not enough to embed in it super-Turing powers. I am always amazed when I hear that a computational model equipped with real numbers allows for hypercomputation. I will start by defining super-Turing power of the scientific presented Universe:

[10] Many people don't really care about clocks. One Professor in may Faculty used to start his lectures in Relativity Theory by saying *Space is measured with rulers, time is measured with clocks, clocks come from Switzerland.* How many of you have listen to lectures about clock functioning?

[11] It is a wonderful exercise to retrieve

$$\vec{F} \;=\; m\,\frac{d^2\vec{r}}{dt^2}$$

out of Schröndinger's equation with $\vec{F}$ given by $-grad\ U$, where U is the classical potential field in the original equation.

THESIS 4 — Up to Turing power, all computations of the Universe are describable by suitable PROGRAMS, which correspond to the prescription by finite means of some rational parameters of the system or some computable real numbers. From Turing power up we have computations that are not describable by finite means: COMPUTATION WITHOUT A PROGRAM. (Gandalf's principle of hidden Universe computation.)

Is it not funny? Computation without a program... When we observe natural phenomena and endow them with computational significance, it is not the algorithm we are observing but the process. Some objects near us may be performing hypercomputation: we observe them, but we will never be able to simulate their behaviour on a computer. What is then the profit of such a theory of computation to Science? The point is that *Gandalf's principle* does not tell us about hyper-machines. In this sense hypercomputation can exist. I presume that most of the reactions of the scientific community against hypercomputation are more related with the *crazy* idea of building a hypermachine. I think it is also one of the sources of criticism against the work of Hava Siegelmann and Eduardo Sontag in [38,39]

But to help the reader to understand that the real numbers are not enough to produce any kind of hypercomputation we call Analog Computation for the rescue.

In the 1940s, two different views of the brain and the computer were equally important. One was the analog technology and theory that had emerged before the war. The other was the digital technology and theory that was to become the main paradigm of computation. [12] The outcome of the contest between these two competing views derived from technological and epistemological arguments. While digital technology was improving dramatically, the technology of analog machines had already reached a significant level of development. In particular, digital technology offered a more effective way to control the precision of calculations. But the epistemological discussion was, at the time, equally relevant. For the supporters of the analog computer, the digital model — which can only process information transformed and coded in binary — wouldn't be suitable to represent certain kinds of continuous variation that help determine brain functions. With analog machines, on the contrary, there would be few or no steps between natural objects and the work and structure of computation (cf. [25,14]). The 1942–52 Macy Conferences in cybernetics helped to validate digital theory and logic as legitimate ways to think about the brain and the machine [25]. In particular, those conferences helped made McCulloch-Pitts' digital model of the brain [22] a very influential paradigm.

[12] For example, students at MIT could at that time learn both about differential analyzers and electronic circuits for binary arithmetic [25].

The descriptive strength of McCulloch-Pitts model led von Neumann, among others, to seek identities between the brain and specific kinds of electrical circuitry [14].

Going back to the roots of Analog Computation theory by starting with Claude Shannon's so-called General Purpose Analog Computer (GPAC). [13] This was defined as a mathematical model of an analog device, the Differential Analyzer, the fundamental principles of which were described by Lord Kelvin in 1876 (see [2]). The Differential Analyzer was developed at MIT under the supervision of Vannevar Bush and was indeed built in 1931, and rebuilt with important improvements in 1941. The Differential Analyzer input was the rotation of one or more drive shafts and its output was the rotation of one or more output shafts. The main units were gear boxes and mechanical friction wheel integrators, the latter invented by the Italian scientist Tito Gonella in 1825 ([2]). From the early 1940's, the differential analyzers at Manchester, Philadelphia, Boston, Oslo and Gothenburg, among others, were used to solve problems in engineering, atomic theory, astrophysics, and ballistics, until they were dismantled in the 1950s and 1960s following the advent of electronic analog computers and digital computers ([2,15]). Shannon (in [37]) showed that the GPAC generates the *differentially algebraic functions*, which are unique solutions of polynomial differential equations with arbitrary real coefficients. This set of functions includes simple functions like the exponential and trigonometric functions as well as sums, products, and compositions of these, and solutions of differential equations formed from them. Pour-El in [32] made this proof rigorous.

The fact is that, although GPAC model is physical realizable and is analog model of the Universe, inputting and outputting real numbers, it does not compute more that the Turing machine, in the sense of Computable Analysis. E.g., we can not output, for a specific architecture of integrators, the digits of the *halting number*.

Barry Cooper and Piergiorgio Odifreddi proceed to say that *The association of incomputability with simple chaotic situations is not new. For instance, Georg Kreisel sketched in [20] a collision problem related to the* 3*-body problem as a possible source of incomputability.*

I think that these ideas are indeed conceived in a few theoretical experiences like in [7,42], although they qualitatively require an unbounded amount of energy [14], and for this reason, not for theoretical reasons, they are not implementable. E.g., returning to Kreisel, the pure mathematical model of Newtonian gravitation is probably capable of *encoding the halting problem of Turing machines*. This hint is given by Frank Tipler too in [41], based on constructions

[13] In spite of being called "general", which distinguish it from special purpose analog computing devices, the GPAC is not a uniform model, in the sense of von Neumann.
[14] Although the total amount of energy involved does not change.

similar to Xia's 5-body system (in [42]), were we have two parallel binary systems and one further particle oscillating perpendicularly to both orbits. This particle suffers an infinite number of mechanical events in finite time (e.g., moving back and forth with increasing speed). Can we encode a universal Turing machine in the initial conditions? This is an unsolved mathematical problem. Thus, it may well be that the system of Newtonian mechanics together with the inverse square law is capable of non-Turing computations. The hypercomputation power that this system may have is not coded in any real number but *in its own dynamics*. How to classify such a *Gedanken experiment*?

In the *Billiard Ball Machine model*, proposed by Fredkin and Toffoli in [8], any computation is equivalent to the movement of the balls at a constant speed, except when they are reflected by the rigid walls or they collide (preserving global kinetic energy) with other balls, in which case they *ricochet* according to the standard Newtonian mechanics. The *Billiard Ball Machine* is Universal. Moreover, the faster the balls move, the faster a given computation will be completed.

Newtonian physical systems that perform an infinite number of operations in a finite time are well known. Specifically, we just have to consider 4 point particles moving in a straight line under the action of their mutual gravity. Mather and McGee have shown in [21] that the masses and the initial data of the particles can be adjusted to impress to the particles infinite velocity in finite time. Gerver in [9] published a paper reporting on a model where, using 5 point particles in the plane moving around a triangle, all particles could be sent to infinity in a finite time.

Can these systems encode hypercomputational sets? We aim at obtaining *either a positive or a negative answer to this question*, i.e., (a) either we will be able to prove that *initial conditions do exist* coding for a universal Turing machine, (b) either we are not able to prove such a lower bound but, we will prove that *encoding of input and output exists, together with adjustable parameters coding for finite control* such that we will have an abstract computer inspired by Newtonian gravitation theory. This result, together with a non-computable character of the n-body problem as shown in [40] *inter alia*, will turn to be a *strong basis to discuss a possible Church-Turing thesis' violation.* In fact, the non-computable character of the n-body problem is close to Pour-El and Richards' results [33], and not so close to a mechanical computer rooted in the structure of the inverse square law.

5 Routes to hypercomputation.

Martin Davis published a paper called *The Myth of Hypercomputation* (see [6]) where he introduces the *Davidism* (doctrine in which I have reasons to believe). Martin Davis fight against hypercomputation in [6] is much more related with the *dream* of building a hypermachine. In fact in [39] two branches of such a discipline are opened: first, the route to a hyper-machine that culminates in Hava Siegelmann's paper in Science and which I think, in agreement with Martin, can be misinterpreted, and, second, the theoretical hypercomputation field were we search for neural nets with weights of several types as representatives of diverse computational classes: integer nets are equivalent to finite automata, rational nets equivalent to Turing machines, polynomial time real nets equivalent to polynomial size Boolean circuits — P/poly —, and so on. In the latter interpretation, neural nets are a uniform model to all these classes. And, INDEED nets with real weights are to worth to be investigated since for decades engineers have been using them theoretically to exploit learning models, in the same sense that differential equations in the field of real numbers are used to model Newtonian gravitation. In the former interpretation, philosophical thinking goes towards Martin's considerations. Just because we don't believe, e.g., in a physical constant Ł of SOMETHING (let us called it *Leopold's universal constant*) with the value of the halting number. Because, if such a constant existed, then we could apply Gandalf's principle to see objects around us performing hypercomputation having we not a tool to reproduce it. That would be the case of having hypercomputation as Alchemy, Barry Cooper and Piergiorgio Odifreddi when they start their article [3]:

> *To the average scientist, incomputability in nature must appear as likely as 'action at a distance' must have seemed before the appearance of Newton's "Principia". One might expect expertise in the theory of incomputability — paralleling that of Alchemy in the seventeenth century — to predispose one to an acceptance of such radical new ideas.*

This is precisely the point I wanted to reach. Alchemy ended and Chemistry started when the *scale* was introduced in Alchemy, a quite good interpretation due to Alexander Koyré. How do we measure hypercomputational behaviour? Supposing that we have a physical *stable* constant having the value of the halting number, then if we measure this constant up to, let us say, 250 digits of precision, becoming aware that the program of code 250 halts for every input, how could we verify it? This would work as a call for observational refutation, it would be, like for Leverrier and Adams, a matter of faith, but in this case without Roemer's telescope in Berlin. Hava's paper in Science looks like Leverrier and Adams trying to convince the scientific community that there is an alien out there. Why was the community not convinced? Well, in first place it seems that nothing in computing escapes to mathematical explanation, like Uranus escaped to his computed orbit. But this is not obvious, since sometimes the scientific community do not react as Airy did. Do you remember about the scandalous trial in London in 1877? (I learned this from Michio Kaku's Hyperspace in [19].)

A psychic from U.S.A. visited London and *bent metal objects at a distance.* He was arrested for fraud. Normally, this trial might have gone unnoticed. But eminent physicists came to his defense, claiming that his psychic feats actually proved that he could summon spirits living in the fourth dimension. Many of defenders were Nobel laureates to be. Do you believe that Professor Johann Zollner, from the University of Leipzig came in his defence? William Crookes, do you believe? And what about J. J. Thompson? And Lord Rayleigh? More names?

[Why this difference of attitude? Airy's reaction to the letters of Leverrier and Adams, with mathematical calculations; Thompson and Crookes reaction to *the possibility of psychokinesis* working with Zollner.]

Many references to [1] across the paper put me in the trail of Barry Cooper's philosophy that we found very rich, showing how a computer scientist can flight over the boundaries of his expertize to meet other sciences. Newton's *Sensorium Dei* was a metaphysical tool to understand a system of the world that without the intervention of God would collapse in his center of gravity. Leverrier and Adams made people believe again in Newtonian's system of physics. Departures of computed lunar orbit against observations were explained by Euler. The space is ready for Laplacian demon to remove God from physical space since Mr. de Laplace *ne besoin pas de cette hypothese* to understand the marry-go-round of the heavenly bodies in the sky. However, what Laplace didn't know is that, most probably, although this system is deterministic, it encodes its own unpredictability and its own incomputability. Probably, not even Laplace's demon have such a computer. While discussing with Barry Cooper, we got this answer from him:

> *... it seems to me that recursion theorists have not until recently really understood or cared what their subject is about, and most still resist even thinking about it (and maybe the same can be said about complexity theorists...). Actually, Gandy was interesting to talk to — as is Martin Davis, of course. I think it is hard for people of my generation and before to adjust to the new fluidity of thinking (or maybe I should say the old fluidity of thinking of the inter-war years).*

The study of hypercomputation should pursue with formal math as any mathematical concept, and not as constrained research as if Mr. Javert was again put in his *pursuite fantastique* of Mr. Jean Valjean.

This essay could well have ended with a beautiful Celt song to the *Oak*, maybe taken from Ray Bradbury's *Secret Tree*, in the center of Stonehenge, at the sunrise of the longest day in mid-summer, looking to the far Heelstone at a distance.

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